Spin-Orbit Correlations

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GPDs: probabilistic interpretation as Fourier transforms of impact parameter dependent PDFs

\[ H(x, 0, -\Delta_{\perp}^2) \rightarrow q(x, b_{\perp}) \]
\[ \tilde{H}(x, 0, -\Delta_{\perp}^2) \rightarrow \Delta q(x, b_{\perp}) \]
\[ E(x, 0, -\Delta_{\perp}^2) \]
\[ \leftrightarrow \perp \text{deformation of unpol. PDFs in } \perp \text{ pol. target} \]
\[ \leftrightarrow \text{physics: orbital motion of the quarks} \]
\[ \leftrightarrow \text{intuitive explanation for SSAs (Sivers)} \]

\[ E_T = 2\tilde{H}_T + E_T \]
\[ \leftrightarrow \perp \text{deformation of } \perp \text{ pol. PDFs in unpol. target} \]
\[ \leftrightarrow \text{correlation between quark angular momentum and quark transversity} \]
\[ \leftrightarrow \text{Boer-Mulders function } h_{1+}^+(x, k_{\perp}) \]
\[ \leftrightarrow \text{Are all Boer-Mulders functions alike?} \]

Summary
Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of $t$, w.r.t. the average momentum fraction $x = \frac{1}{2} (x_i + x_f)$ of the active quark.

\[
\begin{align*}
\int dx H_q(x, \xi, t) &= F_1^q(t) \\
\int dx \tilde{H}_q(x, \xi, t) &= G_A^q(t) \\
\int dx E_q(x, \xi, t) &= F_2^q(t) \\
\int dx \tilde{E}_q(x, \xi, t) &= G_P^q(t),
\end{align*}
\]

- $x_i$ and $x_f$ are the momentum fractions of the quark before and after the momentum transfer.
- $2\xi = x_f - x_i$.

- GPDs can be probed in deeply virtual Compton scattering (DVCS).
Generalized Parton Distributions (GPDs)

formal definition (unpol. quarks):

\[
\int \frac{dx^-}{2\pi} e^{ix^-p^+x} \left< p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \gamma^+ q \left( \frac{x^-}{2} \right) \right| p \right> = H(x, \xi, \Delta^2) \bar{u}(p')\gamma^+ u(p) \\
+ E(x, \xi, \Delta^2) \bar{u}(p') \frac{i\sigma^{+\nu} \Delta^\nu}{2M} u(p)
\]

in the limit of vanishing \( t \) and \( \xi \), the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

\[
H_q(x, 0, 0) = q(x) \quad \tilde{H}_q(x, 0, 0) = \Delta q(x).
\]
### Form Factors vs. GPDs

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$q(x, b_\perp) = \text{impact parameter dependent PDF}$
Impact parameter dependent PDFs

Define localized state [D. Soper, PRD 15, 1141 (1977)]

\[ |p^+, R_\perp = 0_\perp, \lambda \rangle \equiv \mathcal{N} \int d^2 p_\perp |p^+, p_\perp, \lambda \rangle \]

Note: \( \perp \) boosts in IMF form Galilean subgroup \( \Rightarrow \) this state has

\[ R_\perp \equiv \frac{1}{p_\perp} \int dx^- d^2 x_\perp x_\perp T^{++}(x) = \sum_i x_i r_{i, \perp} = 0_\perp \]

(cf.: working in CM frame in nonrel. physics)

Define impact parameter dependent PDF

\[ q(x, b_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, R_\perp = 0_\perp | \bar{q}(-\frac{x^-}{2}, b_\perp) \gamma^+ q(\frac{x^-}{2}, b_\perp) | p^+, R_\perp = 0_\perp \rangle e^{ixp^+x^-} \]

\[ q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} H(x, 0, -\Delta^2_\perp), \]

\[ \Delta q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \tilde{H}(x, 0, -\Delta^2_\perp), \]
Transversely Deformed Distributions and $E(x, 0, -\Delta^2_{\perp})$


- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P+\Delta, \uparrow | \vec{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x, 0, -\Delta^2_{\perp})$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^- x} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x, 0, -\Delta^2_{\perp}).$$

- Consider nucleon polarized in $x$ direction (in IMF)

$$|X\rangle \equiv |p^+, \mathbf{R}_\perp = 0_{\perp}, \uparrow \rangle + |p^+, \mathbf{R}_\perp = 0_{\perp}, \downarrow \rangle.$$  

$\rightarrow$ unpolarized quark distribution for this state:

$$q(x, b_{\perp}) = \mathcal{H}(x, b_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} E(x, 0, -\Delta^2_{\perp}) e^{-ib_{\perp} \cdot \Delta_{\perp}}$$

- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from $j^3$!  

[X.Ji, PRL 91, 062001 (2003)]
Intuitive connection with $\vec{L}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

- $j^+$ larger than $j^0$ when quarks move towards the $\gamma^*$; suppressed when they move away from $\gamma^*$

- For quarks with positive orbital angular momentum in $\hat{x}$-direction, $j^z$ is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

- Details of $\perp$ deformation described by $E_q(x, 0, -\Delta^2_\perp)$

- Not surprising that $E_q(x, 0, -\Delta^2_\perp)$ enters Ji relation!

$$\langle J^i_q \rangle = S^i \int dx \left[ H_q(x, 0, 0) + E_q(x, 0, 0) \right] x.$$
Transversely Deformed PDFs and $E(x, 0, -\Delta^2_{\perp})$

- $q(x, b_{\perp})$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons!

- Mean $\perp$ deformation of flavor $q$ ($\perp$ flavor dipole moment)

\[
d_y^q \equiv \int dx \int d^2b_{\perp} q(x, b_{\perp}) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa^p_q}{2M}
\]

with $\kappa^p_{u/d} \equiv F^{u/d}_2(0) = \mathcal{O}(1 - 2) \Rightarrow d_y^q = \mathcal{O}(0.2 \text{fm})$

- Simple model: for simplicity, make ansatz where $E_q \propto H_q$

\[
E_u(x, 0, -\Delta^2_{\perp}) = \frac{\kappa^p_u}{2} H_u(x, 0, -\Delta^2_{\perp})
\]
\[
E_d(x, 0, -\Delta^2_{\perp}) = \kappa^p_d H_d(x, 0, -\Delta^2_{\perp})
\]

with $\kappa^p_u = 2\kappa_p + \kappa_n = 1.673 \quad \kappa^p_d = 2\kappa_n + \kappa_p = -2.033$.

- Model too simple but illustrates that anticipated deformation is very significant since $\kappa_u$ and $\kappa_d$ known to be large!
SSAs in SIDIS ($\gamma + p \uparrow \rightarrow \pi^+ + X$)

- Use factorization (high energies) to express momentum distribution of outgoing $\pi^+$ as convolution of
  - Momentum distribution of quarks in nucleon
  - Unintegrated parton density $f_{q/p}(x, k_\perp)$
  - Momentum distribution of $\pi^+$ in jet created by leading quark $q$
  - Fragmentation function $D_{q}^{\pi^+}(z, p_\perp)$

- Average $\perp$ momentum of pions obtained as sum of
  - Average $k_\perp$ of quarks in nucleon (Sivers effect)
  - Average $p_\perp$ of pions in quark-jet (Collins effect)
**GPD ↔ SSA (Sivers)**

**Sivers**: distribution of unpol. quarks in \( \perp \) pol. proton

\[
f_{q/p^\uparrow}(x, k_\perp) = f_1^q(x, k_{\perp}^2) - f_{1T}^q(x, k_{\perp}^2) \frac{(\hat{P} \times k_\perp) \cdot S}{M}
\]

- without FSI, \( \langle k_\perp \rangle = 0 \), i.e. \( f_{1T}^q(x, k_{\perp}^2) = 0 \)
- with FSI, \( \langle k_\perp \rangle \neq 0 \) (Brodsky, Hwang, Schmidt)
- FSI formally included by appropriate choice of [Wilson line](https://en.wikipedia.org/wiki/Wilson_loop) gauge links in gauge invariant def. of \( f_{q/p}(x, k_\perp) \)
Why interesting?

- Single spin asymmetry involves nucleon helicity flip
- Quark density chirally even (no quark helicity flip)
- ‘Helicity mismatch’ requires orbital angular momentum (OAM)
- (like $\kappa$), Sivers requires matrix elements between wave function components that differ by one unit of OAM (Brodsky, Diehl, ..)
- Sivers requires nontrivial final state interaction phases
- Sensitive to space-time structure of hadrons
Naively (time-reversal invariance) \( f(x, k_\perp) = f(x, -k_\perp) \)

However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)

Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

\[
f(x, k_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{i p \cdot \xi} \left\langle P, S \left| \bar{q}(0) U_{[0, \infty]} \gamma^+ U_{[\infty, \xi]} q(\xi) \right| P, S \right\rangle \Big|_{\xi^+=0}
\]

with \( U_{[0, \infty]} = P \exp \left( ig \int_0^\infty d\eta^- A^+(\eta) \right) \)

Wilson line phase embodies the FSI from the spectators on the active quark
\[ f_{1T}^+(x, k_\perp)_{DY} = -f_{1T}^+(x, k_\perp)_{SIDIS} \]

- Time reversal: FSI ↔ ISI

**SIDIS:** compare FSI for ‘red’ \( q \) that is being knocked out with ISI for an anti-red \( \bar{q} \) that is about to annihilate that bound \( q \)

- FSI for knocked out \( q \) is attractive

**DY:** nucleon is color singlet → when to-be-annihilated \( q \) is ‘red’, the spectators must be anti-red

- ISI with spectators is repulsive
Single-Spin Asymmetry (Sivers)

- treat FSI to lowest order in $g$

$$\langle k_{q}^{i} \rangle = -\frac{g}{4p^{+}} \int \frac{d^{2}b_{\perp}}{2\pi} \frac{b^{i}}{|b_{\perp}|^{2}} \left\langle p, s \left| \bar{q}(0)\gamma^{+}\frac{\lambda_{a}}{2}q(0)\rho_{a}(b_{\perp}) \right| p, s \right\rangle$$

with $\rho_{a}(b_{\perp}) = \int dr^{-} \rho_{a}(r^{-}, b_{\perp})$ summed over all quarks and gluons

- SSA related to dipole moment of density-density correlations

- GPDs (N polarized in $+\hat{x}$ direction): $u \rightarrow +\hat{y}$ and $d \rightarrow -\hat{y}$

- expect density density correlation to show same asymmetry

$$\langle b_{y} \bar{u}(0)\gamma^{+}\frac{\lambda_{a}}{2}u(0)\rho_{a}(b_{\perp}) \rangle > 0$$

- sign of SSA opposite to sign of distortion in position space
example: $\gamma p \rightarrow \pi X$

$u, d$ distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space ($T$-even!); sign “determined” by $\kappa_u$ & $\kappa_d$

attractive FSI deflects active quark towards the center of momentum

$\rightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\rightarrow$ correlation between sign of $\kappa^p_q$ and sign of SSA: $f^{\perp q}_{1T} \sim -\kappa^p_q$

$f^{\perp q}_{1T} \sim -\kappa^p_q$ confirmed by HERMES results (also consistent with COMPASS $f^{\perp u}_{1T} + f^{\perp d}_{1T} \approx 0$)
\[ f_{1T}^u + f_{1T}^d \approx 0 \] also consistent with sum rule

\[ \int dx \sum_{i \epsilon q,g} f_{1T}^q(x, k_\perp) k_\perp^2 = 0. \]

non-trivial sum rule, not a trivial consequence of momentum conservation (cf. Schäfer Teryaev sum rule for fragmentation) as it does not involve a summation over the whole final state, but only over active partons
Chirally Odd GPDs

\[ \int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \right| \bar{q} \left( -\frac{x^-}{2} \right) \sigma^+ j \gamma_5 q \left( \frac{x^-}{2} \right) \right\rangle p = H_T \bar{u} \sigma^+ j \gamma_5 u + \tilde{H}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} \Delta_\alpha P_\beta}{M^2} u \\
+ E_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T \bar{u} \frac{\varepsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M} u \]

- See also M.Diehl+P.Hägler, hep-ph/0504175.
- Fourier trafo of \( \tilde{E}_T^q \equiv 2\tilde{H}_T^q + E_T^q \) for \( \xi = 0 \) describes distribution of transversity for unpolarized target in \( \perp \) plane

\[ q^i(x, b_\perp) = \frac{\varepsilon^{ij}}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{ib_\perp \cdot \Delta_\perp} \tilde{E}_T^q(x, 0, -\Delta_\perp^2) \]

- origin: correlation between quark spin (i.e. transversity) and angular momentum
Transversity Distribution in Unpolarized Target
Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry
- e.g. quarks at negative $b_x$ with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction
- (qualitative) connection between Boer-Mulders function $h_1^\perp(x, k_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_1^{\perp T}(x, k_\perp)$ and the GPD $E$.

Boer-Mulders: distribution of $\perp$ pol. quarks in unpol. proton

$$f_{q^\uparrow/p}(x, k_\perp) = \frac{1}{2} \left[ f_1^q(x, k_\perp^2) - h_1^\perp q(x, k_\perp^2) \frac{(\hat{P} \times k_\perp) \cdot S_q}{M} \right]$$

$h_1^\perp q(x, k_\perp^2)$ can be probed in DY (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation
probing BM function in tagged SIDIS

- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter
  \[ \leftrightarrow \text{(attractive) FSI provides correlation between quark spin and } \perp \text{ quark momentum } \Rightarrow \text{ BM function} \]
- Collins effect: left-right asymmetry of \( \pi \) distribution in fragmentation of \( \perp \) polarized quark \( \Rightarrow \) ‘tag’ quark spin
  \[ \leftrightarrow \cos(2\phi) \text{ modulation of } \pi \text{ distribution relative to lepton scattering plane} \]
  \[ \leftrightarrow \cos(2\phi) \text{ asymmetry proportional to: } \text{Collins } \times \text{ BM} \]
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution

⊥ quark pol.
QED: when the $\gamma^*$ scatters off $\perp$ polarized quark, the $\perp$ polarization gets modified
- gets reduced in size
- gets tilted symmetrically w.r.t. normal of the scattering plane

quark pol. before $\gamma^*$ absorption

quark pol. after $\gamma^*$ absorption

lepton scattering plane
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution

→ ⊥ quark pol.
Quark Transversity Distribution after $\gamma^*$ absorption

$\rightarrow \perp$ quark pol.

quark transversity component in lepton scattering plane flips
probing BM function in tagged SIDIS

$\perp$ momentum due to FSI

$\rightarrow \perp$ quark pol.

$\downarrow \mathbf{k}_\perp^q$ due to FSI

lepton scattering plane

on average, FSI deflects quarks towards the center
Collins effect

- When a $\perp$ polarized struck quark fragments, the structure of jet is sensitive to polarization of quark.

- Distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric.

- Asymmetry parameterized by Collins fragmentation function.

- Artru model:
  - Struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with $^3P_0$ ‘vacuum’ quantum numbers.
  - Pion ‘inherits’ OAM in direction of $\perp$ spin of struck quark.
  - Produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up.

- Artru model confirmed by HERMES experiment.

- More precise determination of Collins function under way (BELLE).
probing BM function in tagged SIDIS

$\mathbf{k}_\perp$ due to Collins
$\perp$ momentum due to Collins

$\mathbf{k}^q_\perp$ due to FSI

lepton scattering plane

SSA of $\pi$ in jet emanating from $\perp$ pol. $q$
probing BM function in tagged SIDIS

net $\perp$ momentum (FSI+Collins)

$k^q_\perp$ due to FSI

$\mathbf{k}_\perp$ due to Collins

$\mathbf{k}^q_\perp$ net

lepton scattering plane

$\leftrightarrow$ in this example, enhancement of pions with $\perp$ momenta $\perp$ to lepton plane
probing BM function in tagged SIDIS

\[ \text{net } k^\pi_{\perp} \text{ (FSI + Collins)} \]

\[ \downarrow \text{net } k^q_{\perp} \]

lepton scattering plane

\[ \rightarrow \text{expect enhancement of pions with } \perp \text{ momenta } \perp \text{ to lepton plane} \]
Chirally Odd GPDs (sign)

- LC-wave function representation: matrix element for $\vec{E}_T$ involves quark helicity flip
- $\leftrightarrow$ interference between wave function components that differ by one unit of OAM (e.g. s-p interference)
- $\leftrightarrow$ sign of $\vec{E}_T$ depends on rel. sign between s & p components
- bag model: p-wave from lower component

$$\Psi_m = \begin{pmatrix} if \chi_m \\ -g(\vec{\sigma} \cdot \hat{x})\chi_m \end{pmatrix},$$

(relative sign from free Dirac equation $g = \frac{1}{E} \frac{d}{dr} f$)

- $\vec{E}_T \propto -f \cdot g$. Ground state wave function: $f$ peaked at $r = 0 \Rightarrow \vec{E}_T > 0$
- more general potential model: $\frac{1}{E} \rightarrow \frac{1}{E-V_0(r)+m+V_S(r)}$

$\leftrightarrow$ sign of $\vec{E}_T$ same as in Bag model!
Chirally Odd GPDs: sign (M.B. + Brian Hannafious)

- relativistic constituent model: spin structure from SU(6) wave functions plus “Melosh rotation”
  \[ \bar{E}_T > 0 \] (B.Pasquini et al.)
  - origin of sign: “Melosh rotation” is free Lorentz boost
  - relative sign between upper and lower component same as for free Dirac eq. (bag)

- diquark models: nucleon structure from perturbative splitting of spin \( \frac{1}{2} \) ‘nucleon’ into quark & scalar/a-vector diquark: \( \bar{E}_T > 0 \)
  - origin of sign: interaction between \( q \) and diquark is point-like
  - except when \( q \) & diquark at same point, \( q \) is noninteracting
  - relative sign between upper and lower component same as for free Dirac eq. (bag)

- NJL model (pion): \( \bar{E}_T > 0 \)
  - origin of sign: NJL model also has contact interaction!

- lattice QCD (\( u, d \) in nucleon; pion): \( \bar{E}_T > 0 \) (P.Hägler et al.)
Chirally Odd GPDs (magnitude)

- large $N_C$: $\bar{E}_T^u = \bar{E}_T^d$
- Bag model/potential models: correlation between quark orbit and quark spin same for all quark states (regardless whether $j_z = +\frac{1}{2}$ or $j_z = -\frac{1}{2}$)

$\leftrightarrow$ all quark orbits contribute coherently to $\bar{E}_T$

- compare $E$ (anomalous magnetic moment), where quark orbits with $j_z = +\frac{1}{2}$ and $j_z = -\frac{1}{2}$ contribute with opposite sign

$\leftrightarrow E$, which describes correlation between quark OAM and nucleon spin smaller than $\bar{E}_T$, which describes correlation between quark OAM and quark spin: $\bar{E}_T > |E|$

- potential models: $\bar{E}_T \propto \# of q \quad \Rightarrow \quad \bar{E}_T^u = 2\bar{E}_T^d$

$\leftrightarrow$ expect $2\bar{E}_T^d > \bar{E}_T^u > \bar{E}_T^d$

- all of the above confirmed in LGT calcs. (e.g. P.Hägler et al.)
lowest moment of distribution of unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):
Transversity decomposition of $J_q$

- $J^i = \frac{1}{2} \varepsilon^{ijk} \int d^3x \left[ T^{0j} x^k - T^{0k} x^j \right]$

- $J_q^x$ diagonal in transversity, projected with $\frac{1}{2} (1 \pm \gamma^x \gamma_5)$, i.e. one can decompose

$$J_q^x = J_{q,+\hat{x}}^x + J_{q,-\hat{x}}^x$$

where $J_{q,\pm\hat{x}}^x$ is the contribution (to $J_q^x$) from quarks with positive (negative) transversity

\[\leftrightarrow\] derive relation quantifying the correlation between $\perp$ quark spin and angular momentum [M.B., PRD72, 094020 (2006); PLB639, 462 (2006)]

$$\left\langle J_{q,+\hat{y}}^y \right\rangle = \frac{1}{4} \int dx \left[ H_q^T(x, 0, 0) + \bar{E}_q^T(x, 0, 0) \right] x$$

(note: this relation is \underline{not} a decomposition of $J_q$ into transversity and orbital)
GPDs $\xleftrightarrow{FT} IPDs$ (impact parameter dependent PDFs)

- $E(x, 0, -\Delta^{2}_{\perp}) \rightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- origin for deformation: orbital motion of the quarks
- simple mechanism (attractive FSI) to predict sign of $f_{1T}^{q}$

$$f_{1T}^{u} < 0 \quad f_{1T}^{d} > 0$$

distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_{T}^{q} = 2\bar{H}_{T}^{q} + E_{T}^{q}$
- origin: correlation between orbital motion and spin of the quarks
- attractive FSI $\Rightarrow$ measurement of $h_{1}^{\perp}$ (DY,SIDIS) provides information on $\bar{E}_{T}^{q}$ and hence on spin-orbit correlations

expect:

$$h_{1}^{\perp,q} < 0 \quad |h_{1}^{\perp,q}| > |f_{1T}^{q}|$$
Single Spin Asymmetry (Sivers)

- Naively (time-reversal invariance) \( f(x, k_\perp) = f(x, -k_\perp) \)
- However, including the final state interaction (FSI) results in nonzero asymmetry of the ejected quark! (Brodsky, Hwang, Schmidt)
- Gauge invariant definition requires quark to be connected by gauge link. Choice of path not arbitrary but must be chosen along path of outgoing quark to incorporate FSI

\[
f(x, k_\perp) \propto \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{q}(0) U_{[0, \infty]}^+ U_{[\infty, \xi]} q(\xi) | P, S \rangle |_{\xi^+ = 0}
\]

with \( U_{[0, \infty]} = P \exp \left( ig \int_0^\infty d\eta^- A^+(\eta) \right) \)
Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

\[ U_{[0,\infty]} = P \exp \left( ig \int_0^\infty d\eta^- A^+(\eta) \right) = 1 \]

* Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x, k_{\perp})$ requires additional gauge link at $x^- = \infty$

\[
f(x, k_{\perp}) = \int \frac{dy^- d^2y_{\perp}}{16\pi^3} e^{-ixp^+ y^- + i k_{\perp} \cdot y_{\perp}} \times \langle p, s \left| \bar{q}(y) \gamma^+ U_{[y^-, y_{\perp}; \infty^-, y_{\perp}]} U_{[\infty^-, o_{\perp}; 0^-, o_{\perp}]} q(0) \right| p, s \rangle.
\]

[back]