Lattice QCD calculation for NΞ interaction

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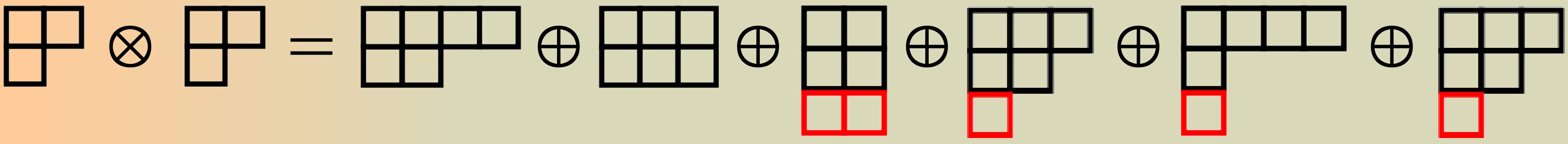

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be,
- The project at J-PARC:
  - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

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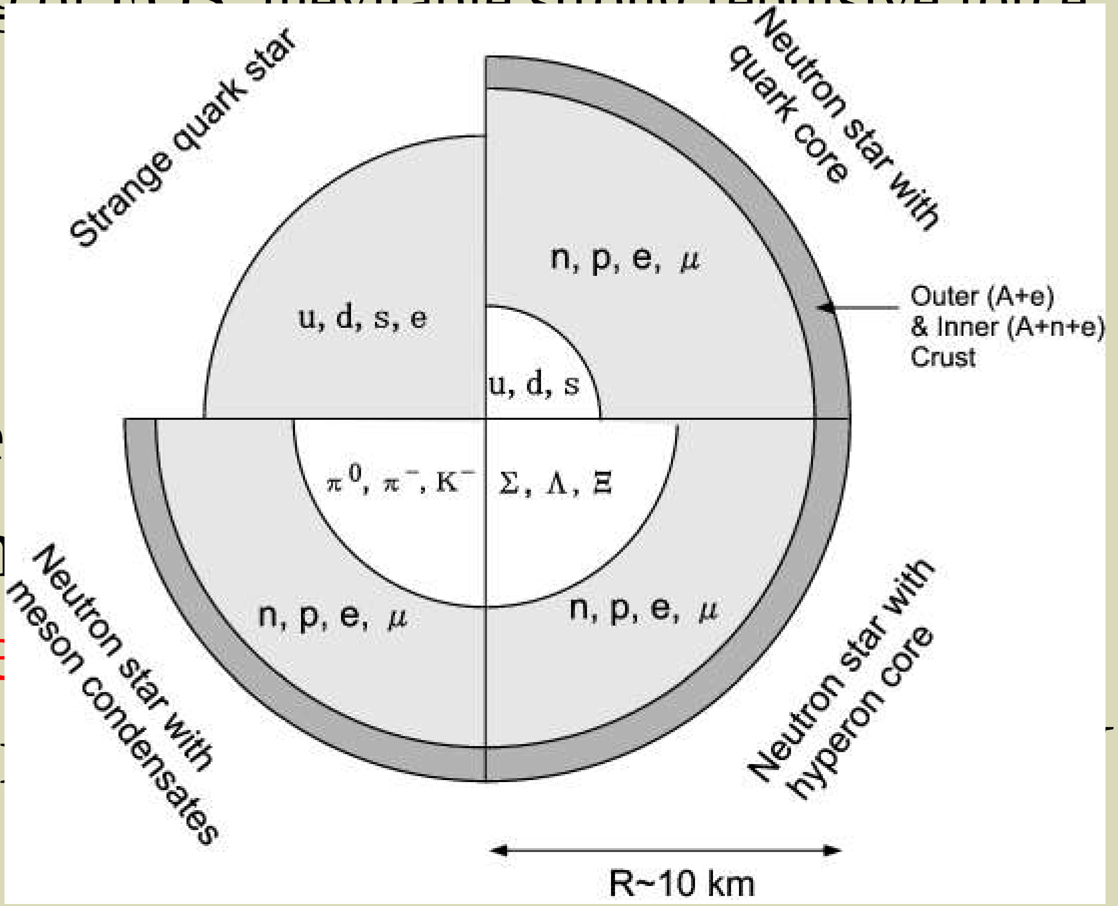

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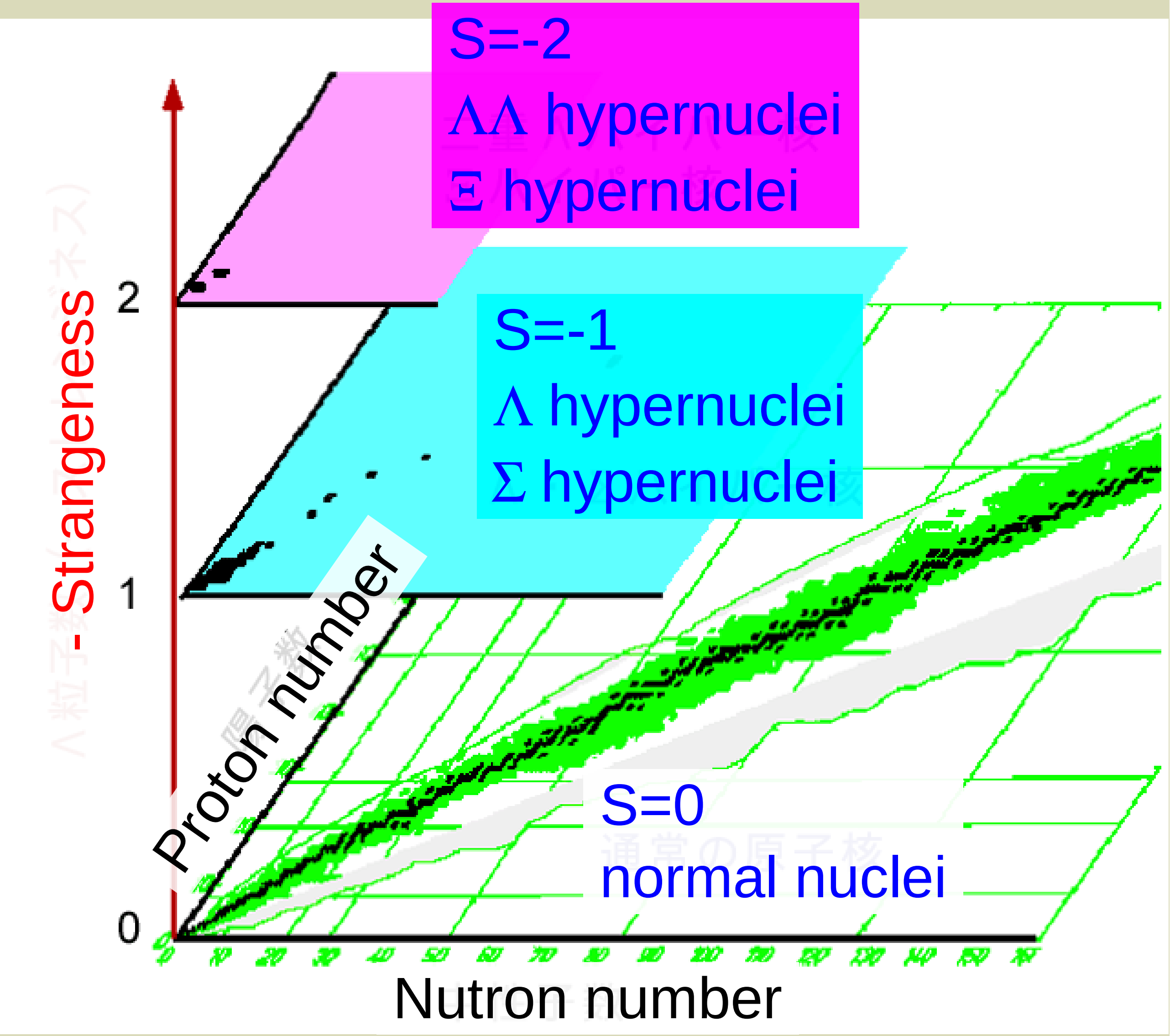
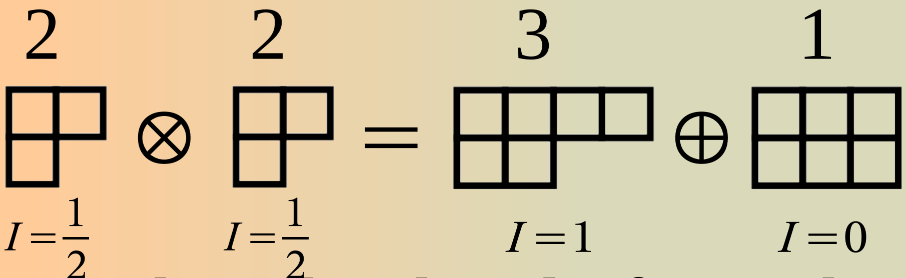
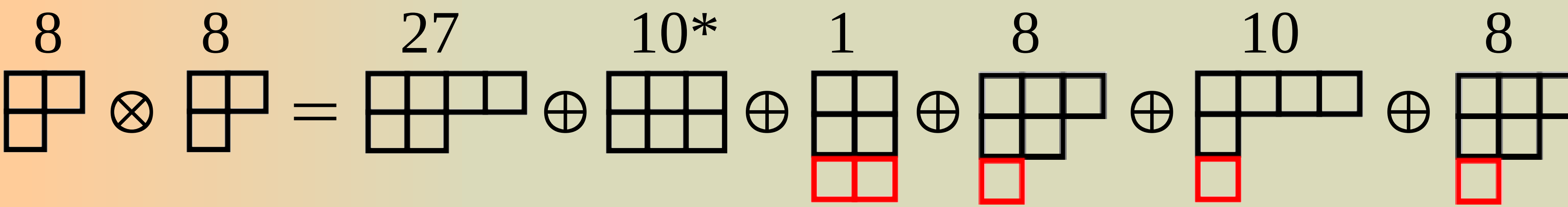


Extension from NN to YN and YY:

- If we take only non-strange sector,
  there are only 2 representations for isospin space.



- On the other hand, if we take account of strange degree of free-dom, other representations should be included.



- This means that the *YN* and *YY* interactions cannot be determined from the precise *NN* experimental data even if we assume the flavor SU(3) symmetry.
- Lattice QCD is desirable for the study of the *YN* and *YY* interac-tion, because this is *ab initio* numerical simulation.

Recent impressive works of lattice QCD:

- S. Aoki, *et al*., PRD**71**, 094504 (2005);

  π-π scattering length from the wave function.

- N. Ishii, *et al*., PRL**99**, 022001 (2007); nucl-th/0611096;

  NN potential from the wave function.

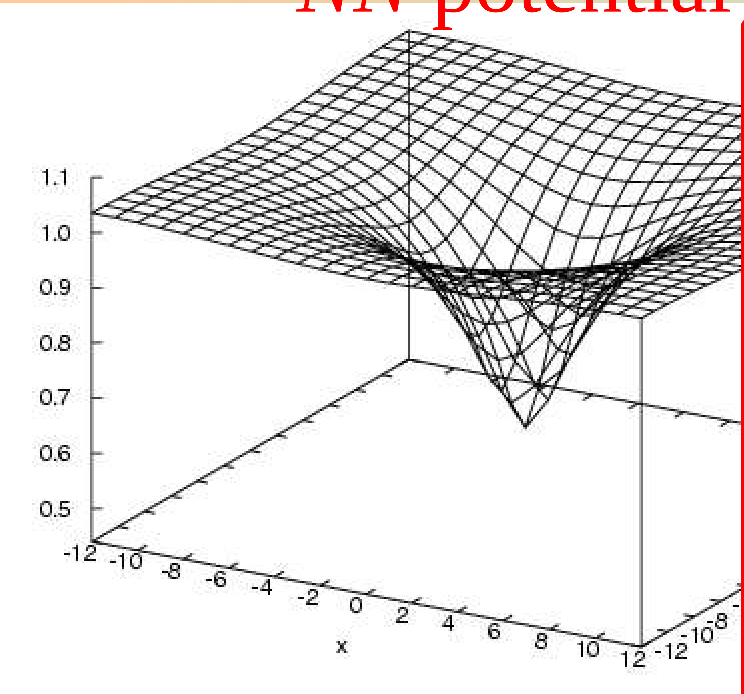
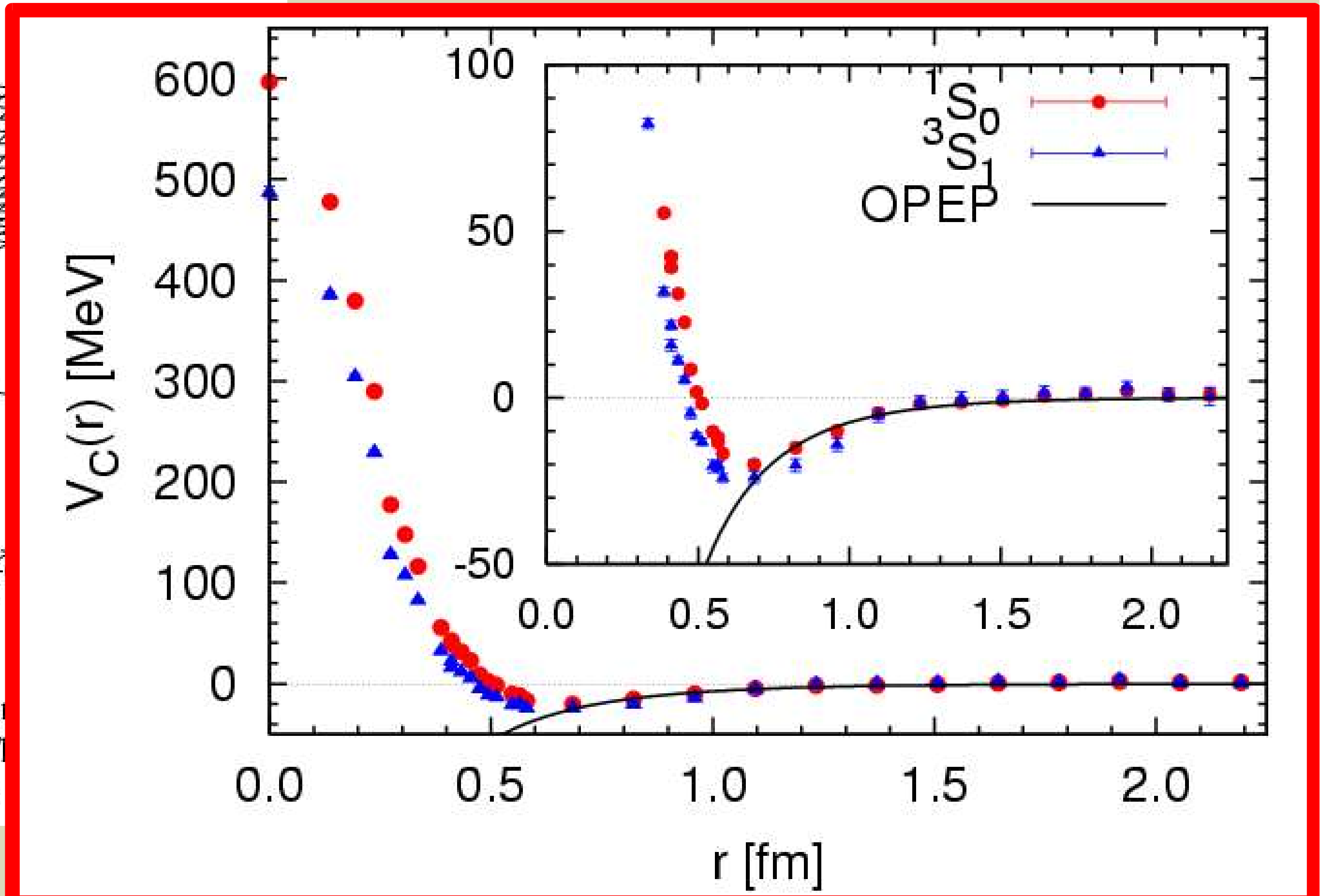


FIG. 1. Two-pion wave function $\phi(\vec{x};k)$ o
$(t,z)=(52,0)$ plane for $m_\pi^2 = 0.273\ \mathrm{GeV}^2$. T
tor is set at $\vec{x}_0 = (7,5,2)$ $(x_0 = |\vec{x}_0| = 8.832)$.



- This work;

  YN and *YY* potentials by applying these techniques.

The purpose of this work

- *YN* and *YY* potentials from lattice QCD
  - *N*Λ, *N*Σ, ΛΛ, *N*Ξ, ...
- *N*Ξ potential as a first step
  - Main target of the J-PARC DAY-1 experiment
  - Few experimental information, so far
  - Simpler operator of Ξ field than that of Λ
- Focus on the *I*=1 channel, $^1S_0$, $^3S_1$
  - *I*=1; *N*Ξ-ΛΣ-ΣΣ: *N*Ξ is the lowest state.
  - *I*=0; ΛΛ-*N*Ξ-ΣΣ: *N*Ξ is not the lowest state.
- *I*=0 channel will be studied in the future.

A recipe for NΞ potential:

- See PRL**99**, 022001 (2007) for detail.

- Calculate the 4-point *N*Ξ correlator on the lattice,

$$\phi_{N\Xi}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\Xi^0_\beta(y,t)\overline{\Xi^0_{\beta'}}(0,t_0)\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - Which has the physical meanings of,
    - Create a *N*Ξ state and making imaginary time evolution, in order to have the lowest state of the *N*Ξ system.
    - Take the amplitude φ(***x***–***y***), which can be understood as a wave function of the non-relativistic quantum mechanics.

- Obtain the effective central potential by assuming that the WF is a solution of effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right)\phi(r) = E\phi(r)$$

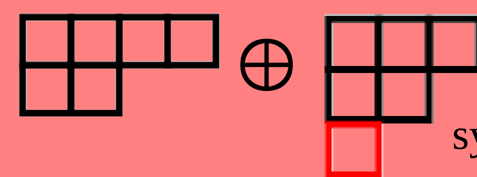
$$V(r) = E + \frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

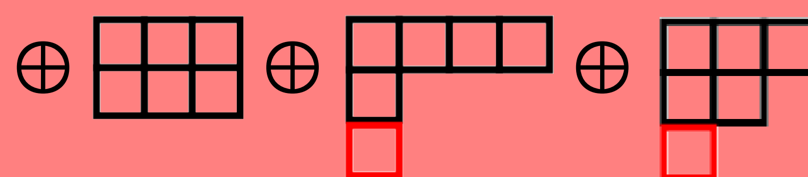
My turn in this work:

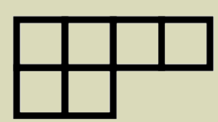
- Calculate the 4-point $N\Xi$ correlator on the lattice,

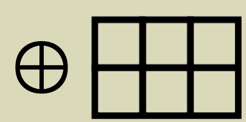
$$\phi_{N\Xi}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\,\Xi^0_\beta(y,t)\,\overline{\Xi^0}_{\beta'}(0,t_0)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

  - This gives the different pattern of the Wick contraction from the *NN*,

$(N\,\Xi) \in$  symmetric for ${}^1S_0$,

$\oplus$  antisymmetric for ${}^3S_1$,

cf. , $(NN) \in$  for ${}^1S_0$,

$\oplus$  for ${}^3S_1$,

- Calculate the 2-point correlators for N and Ξ,

$$\sum_y \langle \Xi^0_\beta(y,t)\,\overline{\Xi^0}_{\beta'}(0,t_0)\rangle \qquad \sum_x \langle p_\alpha(x,t)\,\overline{p_{\alpha'}}(0,t_0)\rangle$$

We need the reduced mass to construct the potential.

$$V(r) = E + \frac{\hbar^2}{2\mu}\frac{\nabla^2\phi(r)}{\phi(r)}$$

Interpolating fields and parameters:

- Interpolating fields:

$$p_{\alpha}(x)=\varepsilon_{abc}(u_a(x)C\gamma_5 d_b(x))u_{c\alpha}(x),$$

$$\Xi^0_{\beta}(y)=\varepsilon_{abc}(u_a(y)C\gamma_5 s_b(y))s_{c\beta}(y),$$

- The lattice calculations were performed by using KEK Blue Gene/L supercomputer.
  - The C++ code reached 1.3GFlops/processor, which is almost a half(46%) of the peak value.

1.3TFlops at 512node que

- Volume: $32^3\times 32$ lattice (L ~ 4.4 fm).
- Lattice spacing: a ~ 0.14 fm.
- Standard Wilson action:
  The main results are obtained with
  - κ_{ud} = 0.1678 for the u and d quarks, and
  - κ_s = 0.1643 for s quark.

Meson masses:
m_π ~ 0.367(1) GeV
m_ρ ~ 0.811(4) GeV
m_K ~ 0.5526(5) GeV
m_{K^*} ~ 0.882(2) GeV

Determination of s quark mass

- In order to determine the strange quark mass (κ_s), we first calculate the meson masses by using six combinations of the parameters;
  - 3 sets for $\kappa_{ud} = \kappa_s$; ← {0.1678, 0.1665, 0.1640},
  - 3 sets for $\kappa_{ud} > \kappa_s$;← {0.1678, 0.1665, 0.1640},
- From the data, We obtain

$$(m_{ps}a)^2 = \frac{B}{2}\left(\frac{1}{\kappa_1} - \frac{1}{\kappa_c}\right) + \frac{B}{2}\left(\frac{1}{\kappa_2} - \frac{1}{\kappa_c}\right)$$

$$(m_v a) = C + \frac{D}{2}\left(\frac{1}{\kappa_1} - \frac{1}{\kappa_c}\right) + \frac{D}{2}\left(\frac{1}{\kappa_2} - \frac{1}{\kappa_c}\right)$$

  - critical quark mass: κ_c = 0.1693,
  - physical quark mass κ_{phys} = 0.1691 from ($m_\pi a/m_\rho a$)=(135/770),
  - lattice scale *a* = 0.1420 fm from the physical ρ meson mass.
  - Strange quark mass: κ_s = 0.1643

    from the physical *K* meson mass (494 MeV).

Results —— hadron masses

- Path integrals for the correlators are performed by using 1283 gauge configurations, so far:

(17 exceptional configurations are not used.)

- Calculated masses (in units of GeV):

m_π	m_ρ	m_K	m_{K^*}
0.367(1)	0.811(4)	0.5526(5)	0.882(2)
m_p	m_Ξ	m_Λ	m_Σ
1.164(7)	1.379(6)	1.263(5)	1.312(6)

- Interpolating fields for Λ and Σ^+:

$$\Lambda_\alpha(x) = \frac{1}{\sqrt{3}} \varepsilon_{abc} \left\{ (d_a C \gamma_5 s_b) u_{c\alpha} + (s_a C \gamma_5 u_b) d_{c\alpha} - 2 (u_a C \gamma_5 d_b) s_{c\alpha} \right\}$$

$$\Sigma_\beta^+(y) = -\varepsilon_{abc} (u_a(y) C \gamma_5 s_b(y)) u_{c\beta}(y),$$

Results —— hadron masses

Note: The present results for the baryon masses provide the correct order of the threshold energies of two baryon states with the strangeness S=-2.

$$E_{th}(\Sigma\Sigma) = 2.624(11)\ \text{GeV}$$

$$E_{th}(\Lambda\Sigma) = 2.575(11)\ \text{GeV}$$

$$E_{th}(N\Xi) = 2.544(12)\ \text{GeV}$$ Present calc (I=1)

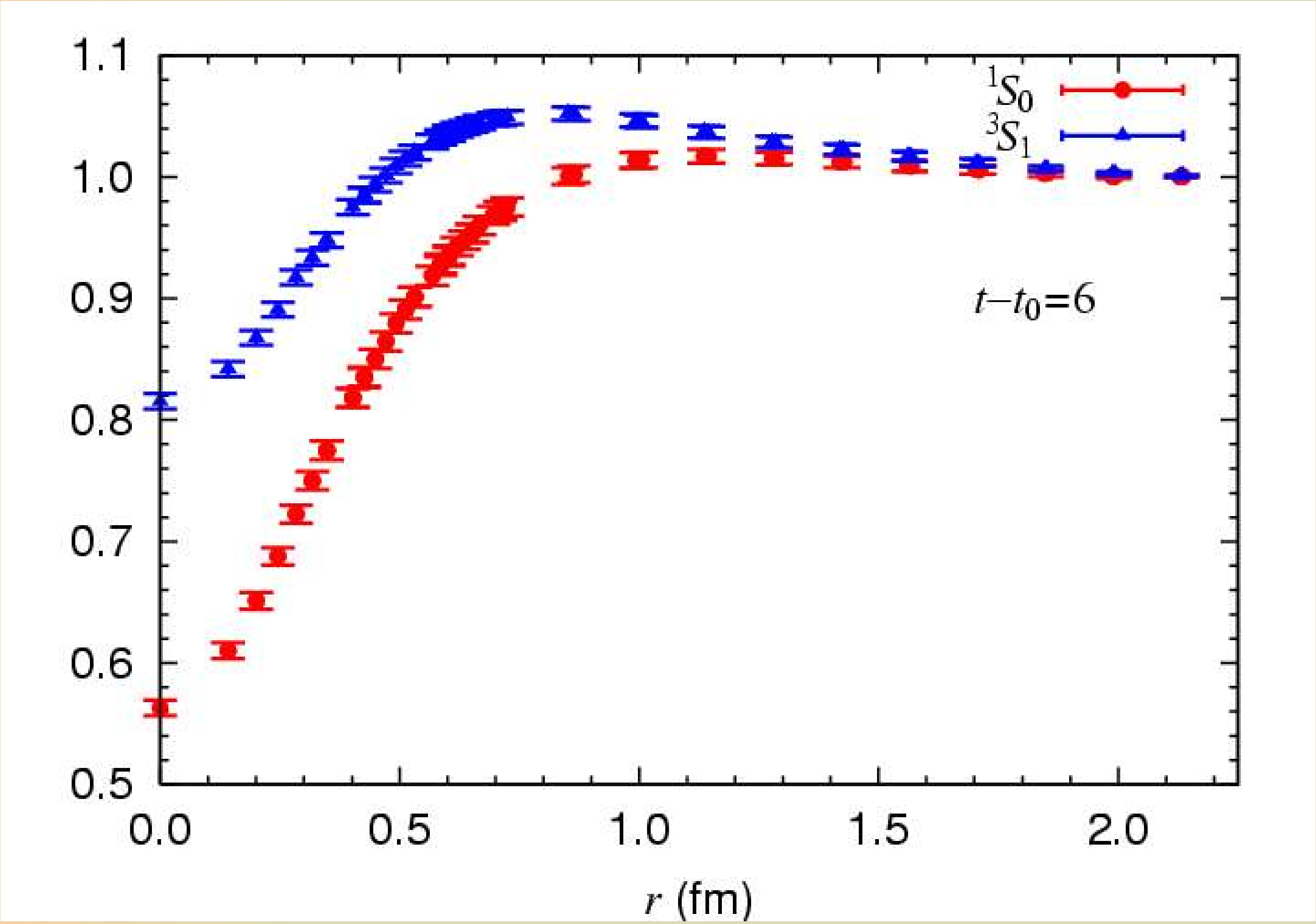
$$E_{th}(\Lambda\Lambda) = 2.525(11)\ \text{GeV}$$

The ΛΛ channel is not allowed in the present case because of isospin conservation.

This maintains the desirable asymptotic behavior of the wave function.

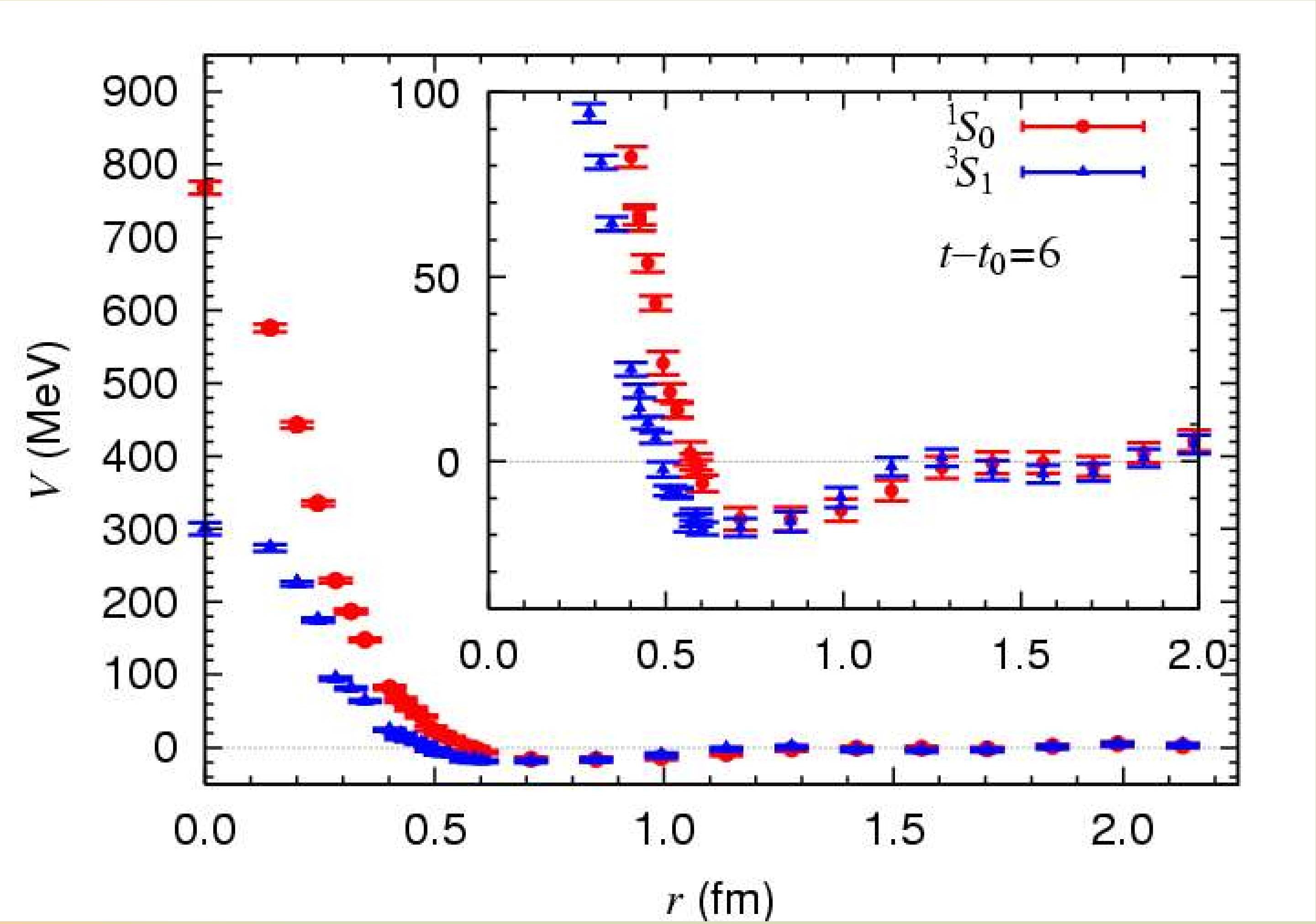
Results —— wave function

- Suggests the repulive core in short range and attractive force in medium range (0.5fm < r < 1fm) for both spin S=0 and 1.



Results —— potential

- NΞ potential (I=1), from lattice QCD for the first time.



- Strong repulsive core in spin S=0 channel.
- Strong spin dependence.

Results —— potential (cont.)

- The net interaction of the NΞ (I=1) is attractive.

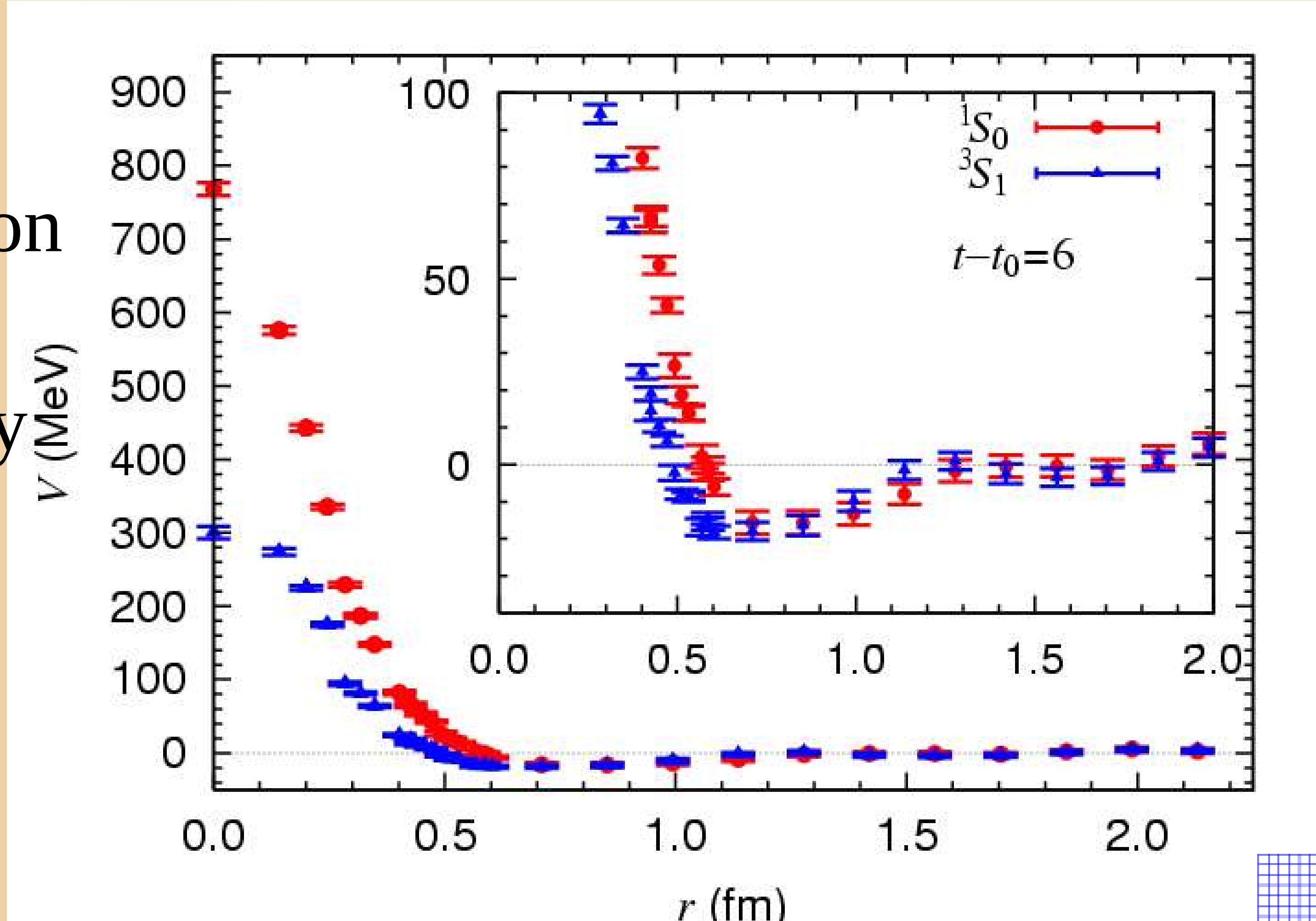
Wave function

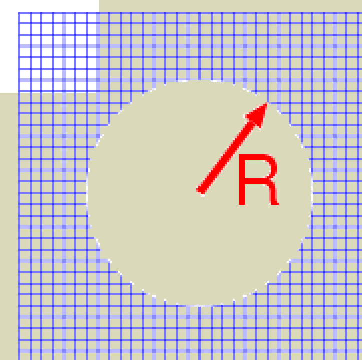
↓

Energy (k^2)

↓

Phase shift





- Scattering lengths:

a_{0s} ~ 0.09 fm ($^1S_0$), a_{0t} ~ 0.2 fm ($^3S_1$).

$$k\cot\delta_0(k) = 1/a_0 + O(k^2)$$

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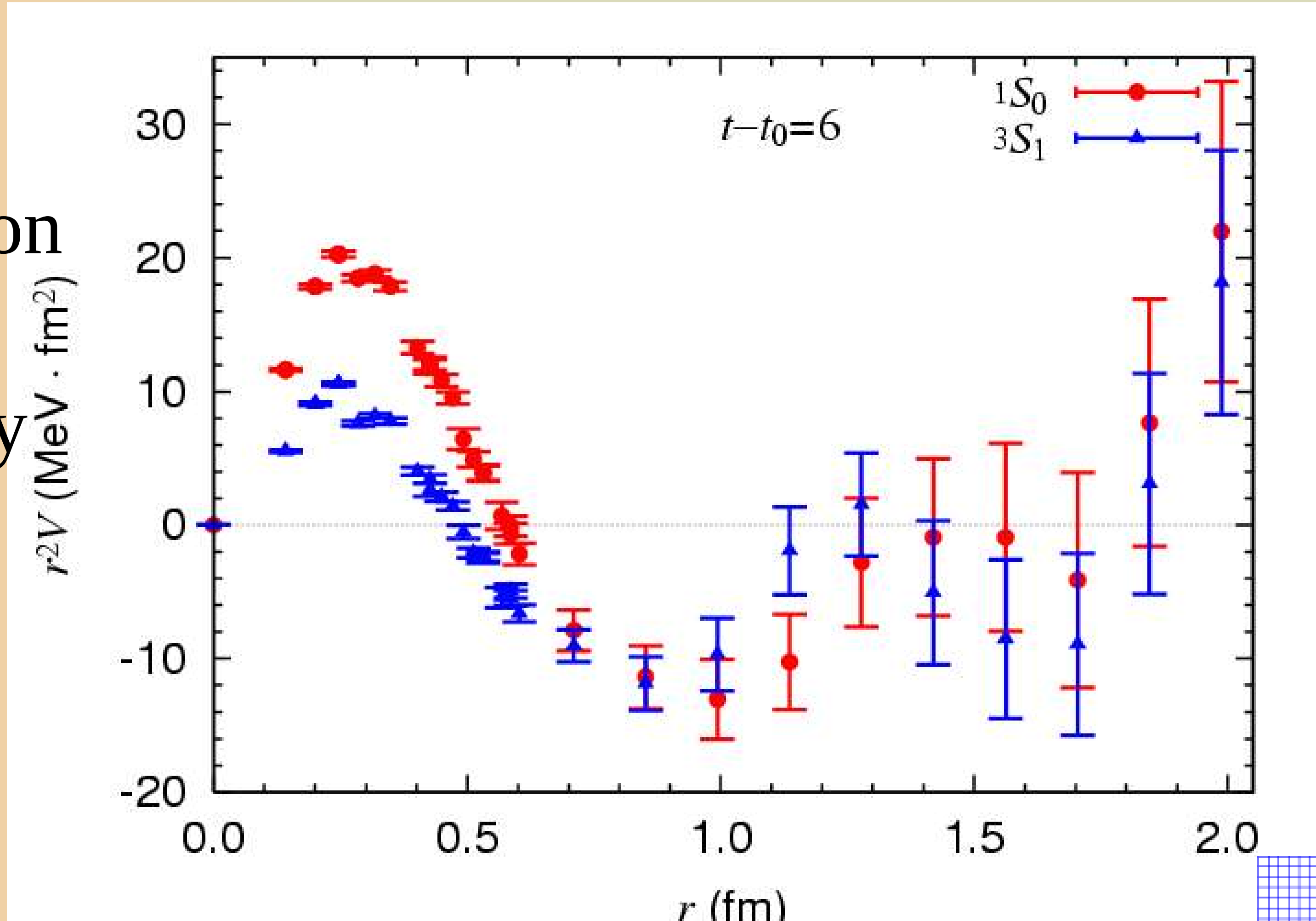

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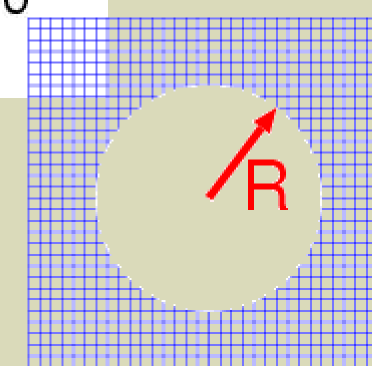

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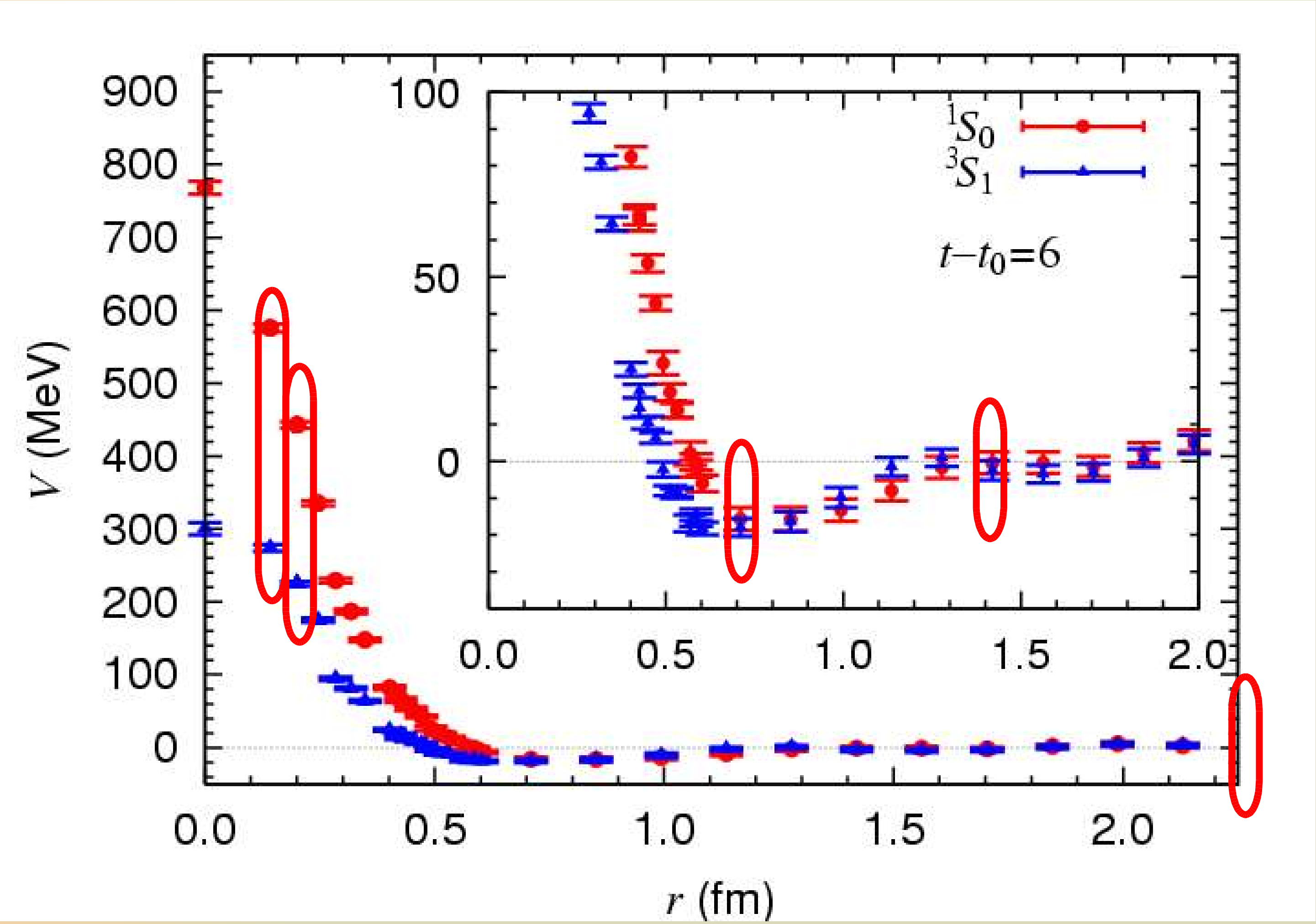
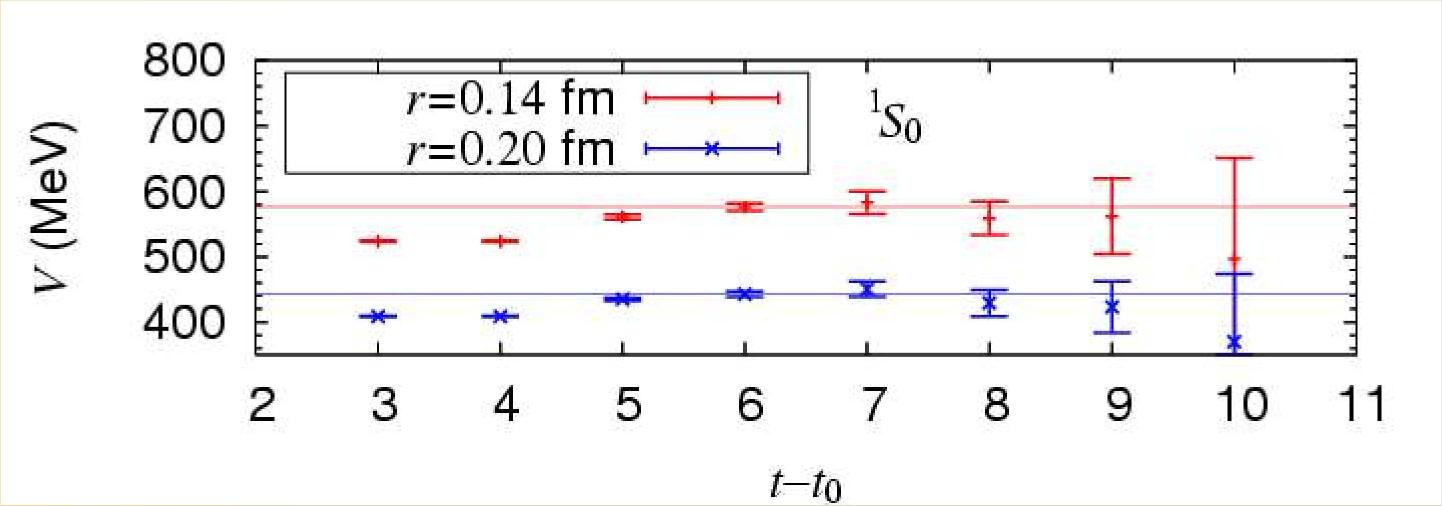
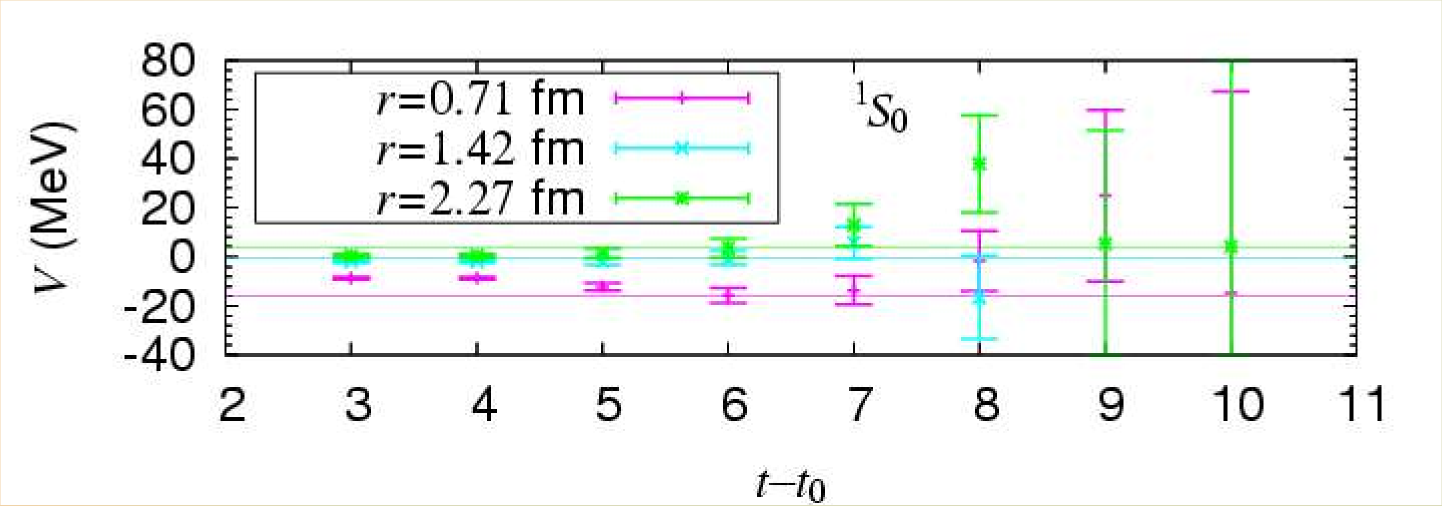


- Strong repulsive core in spin S=0 channel.
- Strong spin dependence.

Results —— potential

- Time-slice dependence of the NΞ potential (I=1, S=0).





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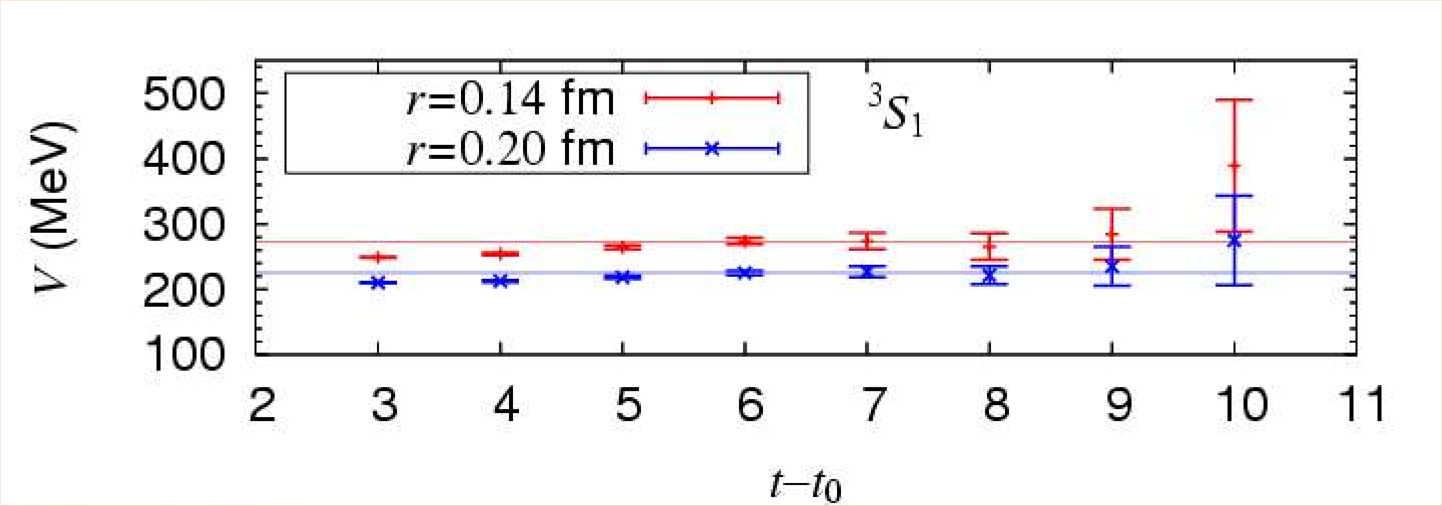

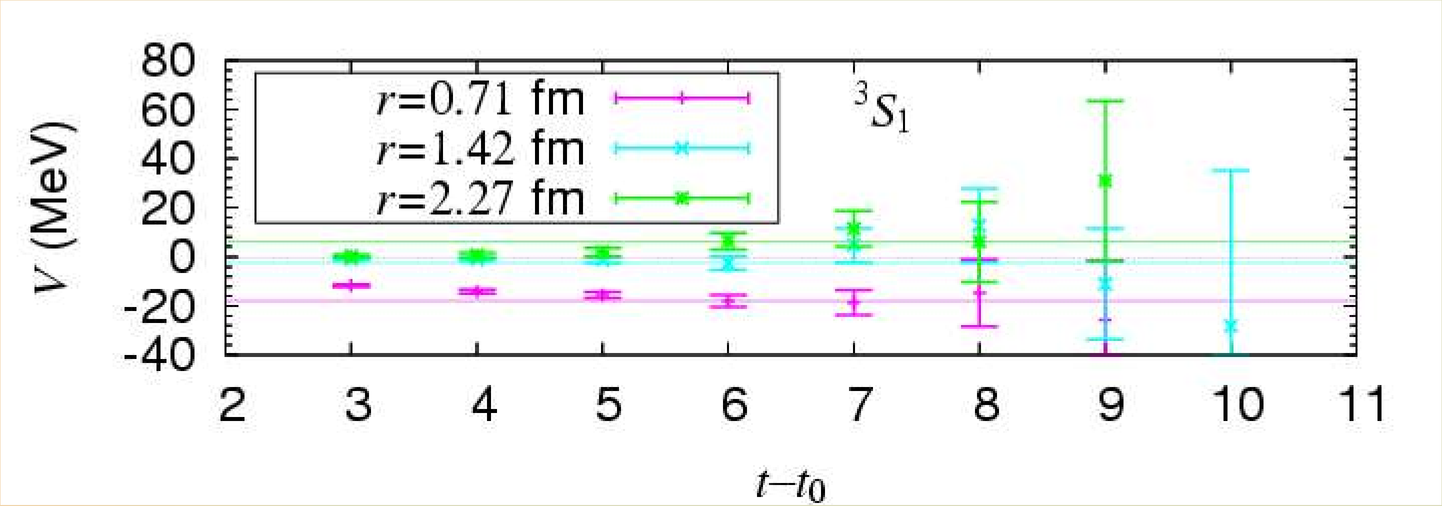

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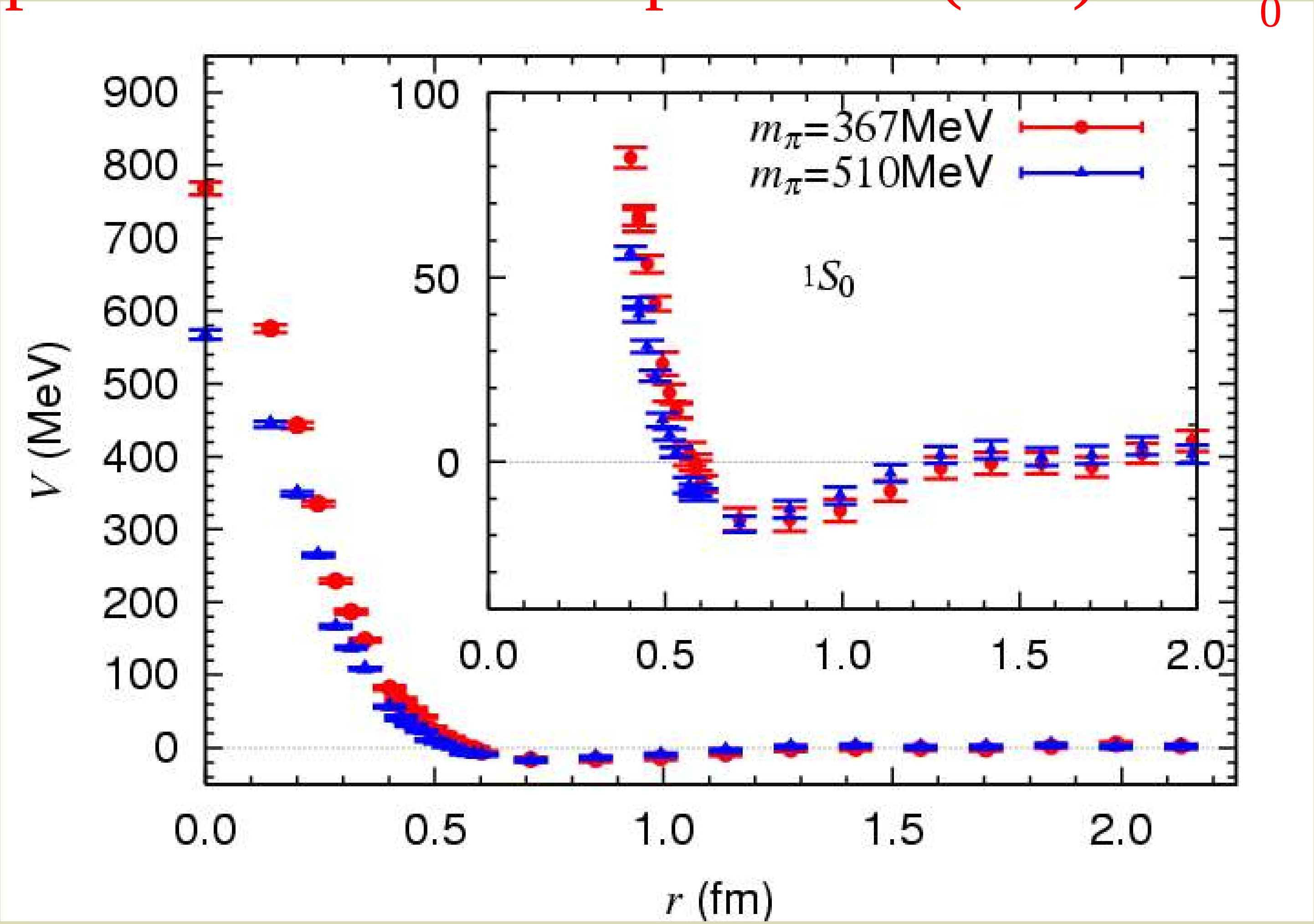
Results —— potential (cont.)

- Quark mass dependence of the NΞ potential (I=1) in $^1S_0$.

  - m_π=367MeV, N_{conf}=1283, t-t_0=6.

  - m_π=510MeV, N_{conf}=1000, t-t_0=7.



- Strength of the repulsive core increases, and
- Interaction rage (slightly) increases,

Results —— potential (cont.)

- Quark mass dependence of the NΞ potential (I=1) in $^3S_1$.
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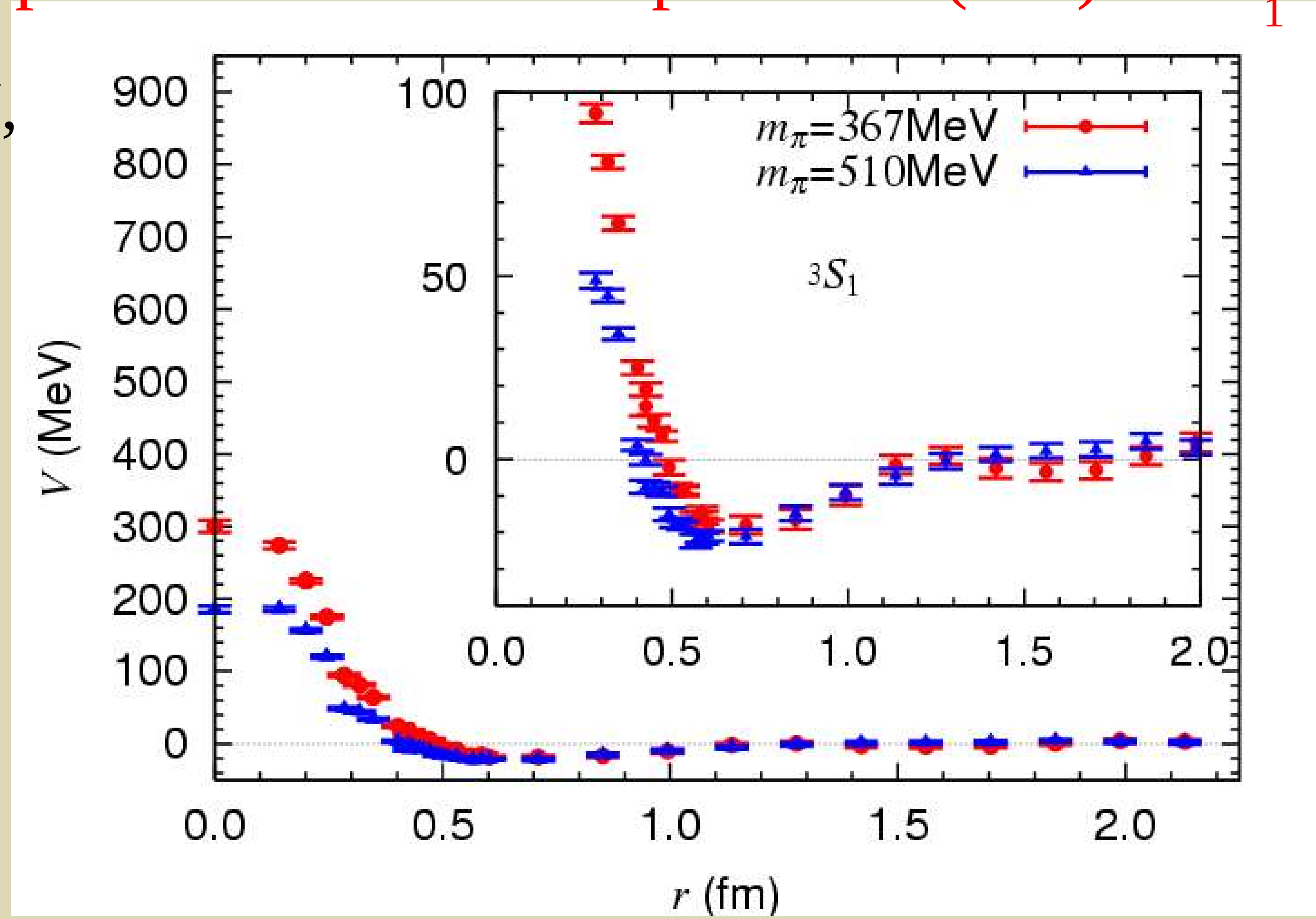

- Strength of the repulsive core increases, too, but
- Interaction rage (little or not) increases,

Summary:

- The first lattice QCD results for *YN* potentials.
- *N*Ξ potential in isospin *I*=1 channel.
  - Which will be studied by DAY-1 experiment at J-PARC.
- Attractive on the whole, and strong spin dependence:
  - Strong repulsive core in spin *S*=0 channel and
  - Relatively weak repulsive core in spin *S*=1 channel.
  - $^3S_1$ channel is more attractive than $^1S_0$ channel.
  - Quark mass dependense.
- We will study further with
  - More detailed analysis such as the *NN* potential.
  - Physical quark mass for *u* and *d* quarks.
  - Beyond the quenched approximation.
  - Other baryon-baryon pairs.