Neutron Interferometry and Search for Non-Newtonian Gravity

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Extra dimensions and Gravity

$$R \sim M^{-1}\left(\frac{M_{Pl}}{M_*}\right)^{2/d} \sim 10^{32/d-17}\,cm$$

$$V_G(r) = -G\frac{mM}{r}\left(1+\alpha_G e^{-r/\lambda}\right)$$

Dark Matter

the "dark energy" density is $\sim(10^{-3}\ eV)^4$

→

sensitivity of neutrons for test of gravity at corresponding distances $(< 0.1\ mm)$

Some characteristic scales (UCN)

$$E_n = 100neV \quad \Rightarrow \quad \lambda = 90.4nm$$

$$E_n = 1neV \quad \Rightarrow \quad \lambda = 904nm$$

Neutron Interferometry

$$\Delta\Phi = \Delta\Phi_{nuclear} + \Delta\Phi_{ne} + \Delta\Phi_{grav}$$

Translation Method

$$V_{eff} = 2\pi G \alpha_G m_n \rho \lambda^2 e^{-x/\lambda}$$ outside the material;

$$V_{eff} = 2\pi G \alpha_G m_n \rho \lambda^2 (2 - e^{-x/\lambda})$$ inside the material.

$$\Delta\Phi = \frac{4\pi G \alpha_G m_n \rho \lambda^3}{k_0} \left(\frac{2m_n}{\hbar^2} \right) \left(1 - e^{-L/\lambda}\right)$$

$$k_0 = \sqrt{\frac{2m_n}{\hbar^2} E_n}$$

Two Plates

$$V_F = \frac{2\pi\hbar^2}{m_n} Nb$$

$$k = \sqrt{\frac{2m_n}{\hbar^2}(E_n - V_F)}$$

$$T_0 = \frac{2k_0 k e^{-ikL}}{2k_0 k \cos(k_0 L) - i(k^2 + k_0^2)\sin(k_0 L)}$$

Two plates + Gravity

- 1st: $$k^2 \to k^2 + 2a^2 - a^2 e^{(x-d)/\lambda}\left[1 - e^{-L/\lambda}\right]$$
- Between: $$k_0{}^2 \to k_0{}^2 + a^2\left(e^{-x/\lambda} + e^{(x-L)/\lambda}\right)$$
- 2nd: $$k^2 \to k^2 + 2a^2 + a^2 e^{(d-x)/\lambda}\left[e^{L/\lambda} - 1\right]$$

$$a^2 = 2\pi G \alpha_G m_n \rho \lambda^2$$

For experimental sensitivity of 10^{-4} *rad* and $\lambda_n = 3\mathring{A}$

$$\alpha_G \lambda^3 \leq 3\times10^{-3} m^3$$

Then, for $\lambda \approx 10$nm: $\alpha_G \leq 3\times10^{21}$

Diffraction

$$b_{coh} = b_N + Z\left[1 - f(q)\right] b_{ne} + f_G(q) b_G$$

- For q=0: $b_{coh} = b_N + b_G$
- Bragg reflection

$$F_{H_1} = \sqrt{32}(b_N + Z[1 - f(H_1)]b_{ne} + f_G(H_1) b_G)$$

$$F_{H_3} = \sqrt{32}(b_N + Z[1 - f(H_3)]b_{ne} + f_G(H_3) b_G)$$

$$b_G = -\frac{2 m_n^2 M G \alpha_G \lambda^2}{\hbar^2}$$

$$f_G(q) = \frac{1}{1+(q\lambda)^2}$$

$$f(H_1) = 0.7526 \qquad f(H_3) = 0.4600$$

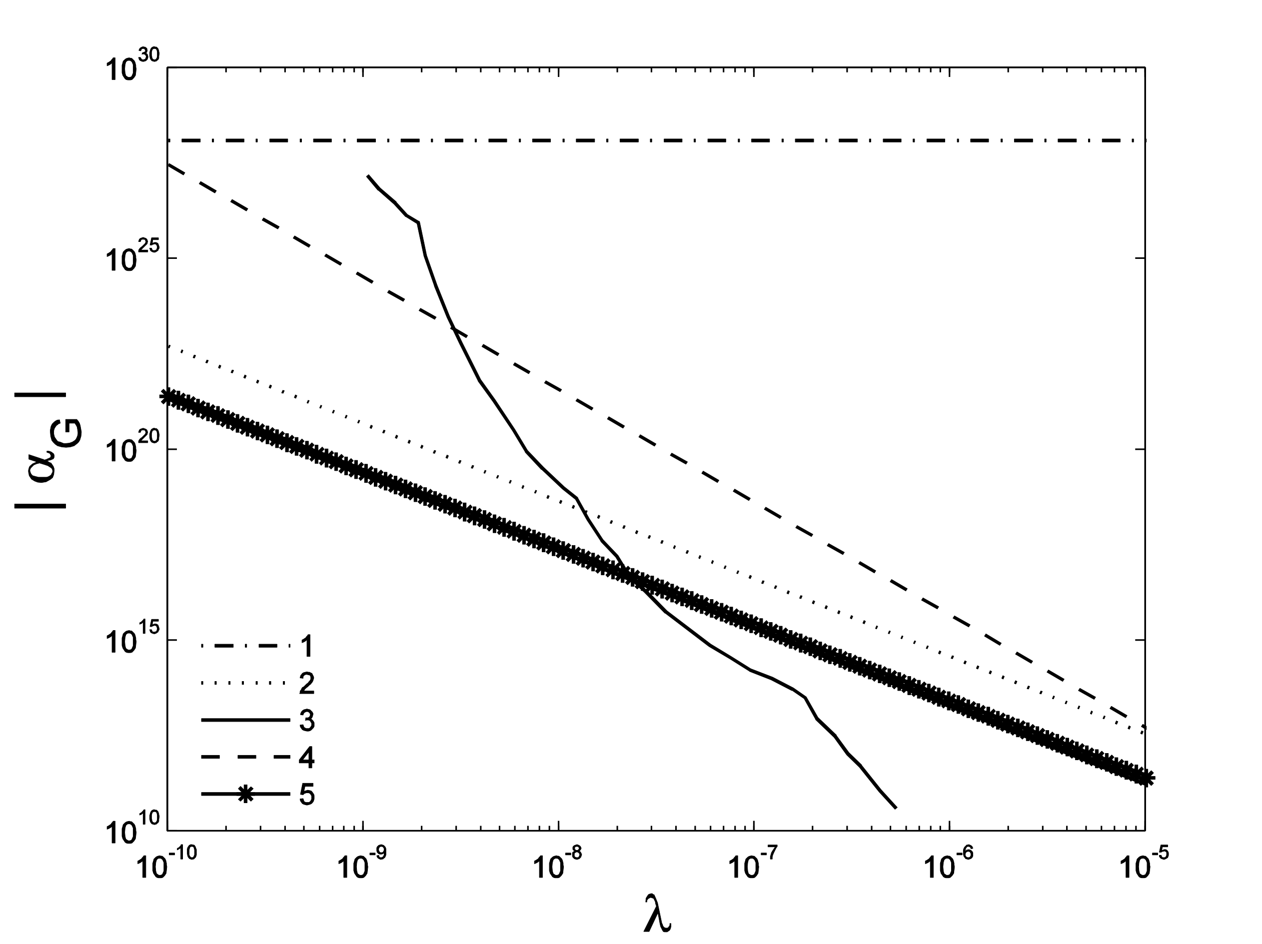
- For $q^{-1} \sim 1\mathring{A}$

$$f_G(q)b_G = -\frac{2m_n^2 MG\alpha_G\lambda^2}{\hbar^2}$$

- For $\lambda \gg 1\mathring{A}$

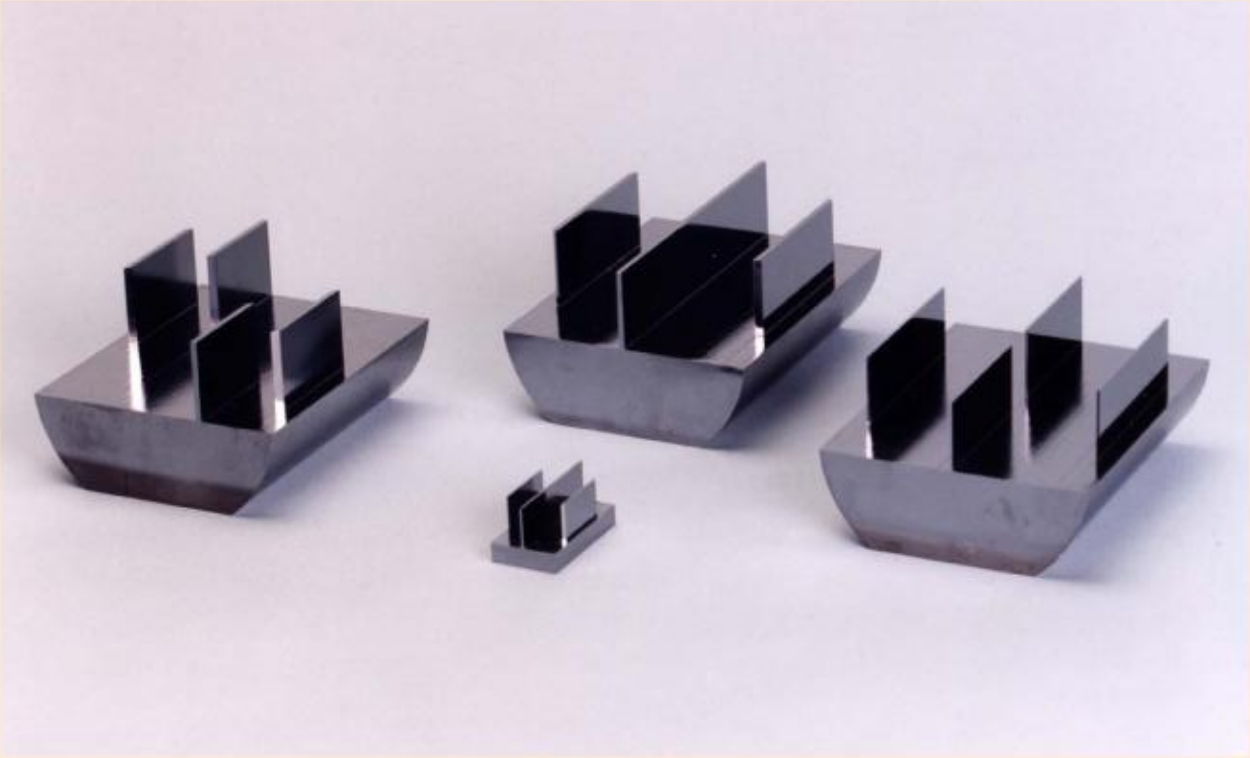
$$f_G(q)b_G \simeq -\frac{2m_n^2 MG\alpha_G}{\hbar^2}\frac{1}{q^2}$$

$$b_G \simeq -1.6\times 10^{-6}(\alpha_G\lambda^2) \qquad \alpha_G\lambda^2 \le 25.6m^2$$

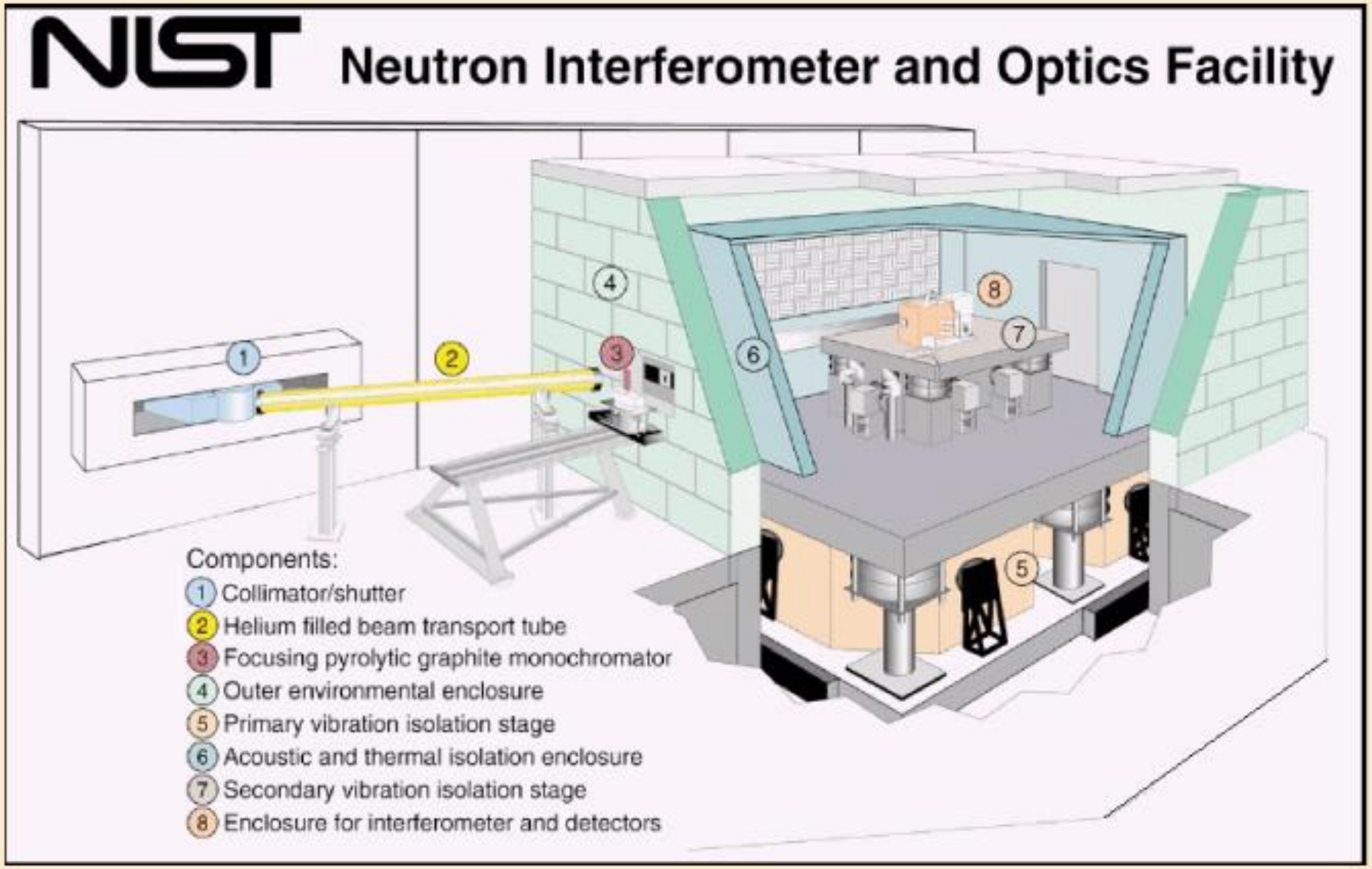


(1) from Nesvizhevsky and Protasov;
(2) from Zimmer and Kaiser;
(3) from the review of Adelberger;
(4) from the "two-plate" method;
(5) from the diffraction method.

NIST perfect crystal silicon interferometers



Courtesy of Fred Weitfeldt



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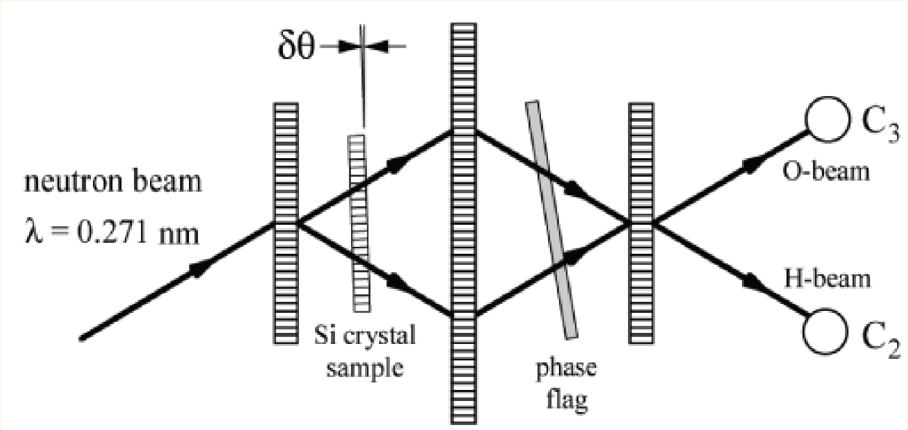
Neutron Interferometer Experiment



off Bragg: $b_{\text{coh}} = b_N + Z[1 - f(0)]b_{ne} = b_N$

near Bragg: $b_{\text{coh}} = b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$

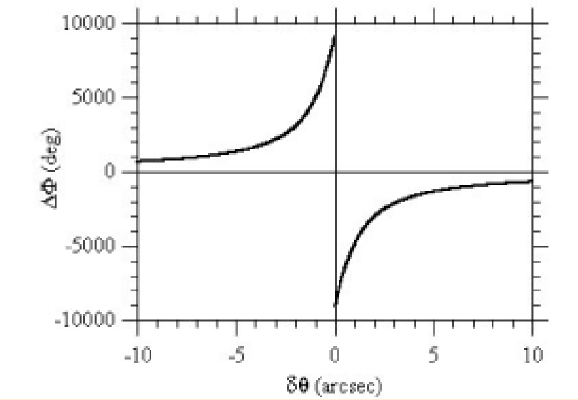
Courtesy of Fred Weitfeldt

Dynamical Phase Shift Through Bragg

$$\Delta\Phi_{\text{dyn}} = \frac{\nu_H}{\cos\theta_B}\left(y \pm \sqrt{1+y^2}\right)D$$

D = crystal thickness

scaled misset angle $y = \dfrac{k\sin 2\theta_B}{2\nu_H}$

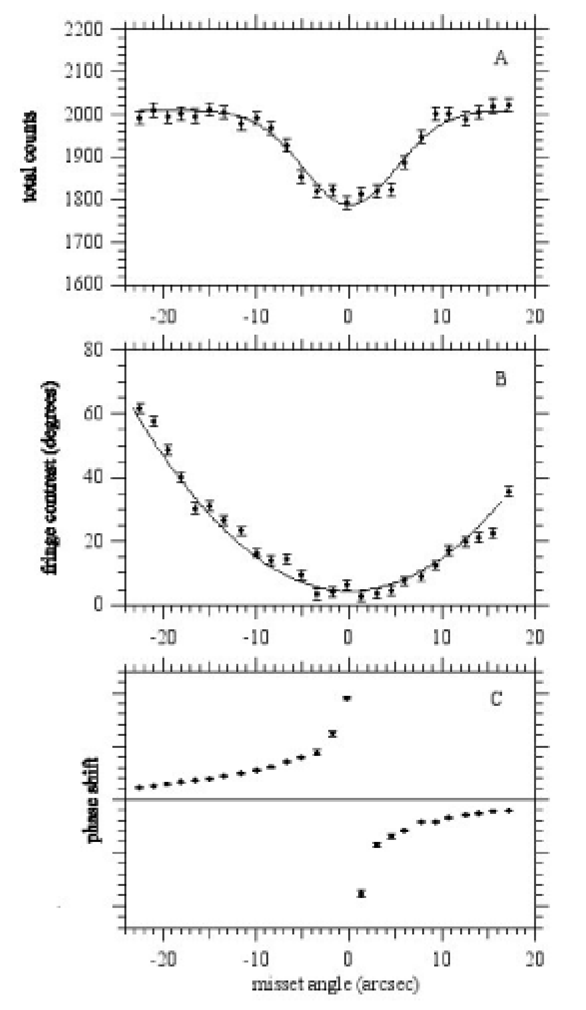


$$\nu_H = \frac{F_{111}\lambda}{V_{\text{cell}}} = \frac{\sqrt{32}\lambda}{V_{\text{cell}}} b_{\text{coh}}$$

near Bragg: $b_{\text{coh}} = b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$

Courtesy of Fred Weitfeldt

Preliminary Data:



These data were taken at NIST in September 2005

Courtesy of Fred Weitfeldt

Two Plates: QM-Solution

Observations:

- solution of Mathieu - type equation
- PR: coherent propagation of neutrinos through materials with density profile variations (Akhmedov, 1999)
- “Harmonic oscillator “= “one-dimensional potential barrier (Pitaevsky)
- Eckart’s potential : the existence of quantum parametric resonance

Quantum parametric resonance

$$d^2\psi(x)/dx^2 + k^2(x)\psi(x) = 0$$

$$k^2(x) = k_0^2(1 + 2\varepsilon\sin((2+\delta)k_0 x)) \quad \text{at} \quad |\varepsilon|,|\delta| \ll 1$$

- L. P. Pitaevsky, A. M. Perelomov, Ya. B. Zeldovich
- (L. D. Landau, E. M. Lifshitz)

Quantum PR (phase)

- The potential in between the slabs: $k_0{}^2(1+\eta\cosh(x/\lambda))$

  where $\eta = 2a^2\exp(-L/2\lambda)/k_0^2$

Since $$\cosh(x/\lambda) = \frac{\sinh(L/\lambda)}{(L/\lambda)} + \sum_{n=1}^{\infty}\left(\frac{2(-1)^n(L/\lambda)\sinh(L/\lambda)}{(L/\lambda)^2+n^2\pi^2}\cos\left(\frac{n\pi x}{L}\right)\right)$$

QPR conditions: $$2k_0/\left(1+\eta\frac{\sinh(L/\lambda)}{(L/\lambda)}\right) = \frac{n\pi}{L}$$

or for very small parameter η: $\lambda_n \simeq 4L/n$

With the width $$\gamma \simeq \frac{a^2\lambda_n^2}{\pi^2}\frac{(L/\lambda)\sinh(L/\lambda)}{(L/\lambda)^2+16\pi^2(L/\lambda_n)^2}e^{-L/(2\lambda)}$$

One-Dim transmission

- that transmission amplitude for exponentially decreasing potentials have infinite number of singularities in the complex momentum plane

$$k = -2i / \lambda$$

- For two overlapping potentials the transmission amplitude has a second order pole at the same position, defined by the slope:

then, the region of maximal sensitivity is: $\lambda_n \leq \lambda$

Toy Model

Two potentials: one is a localized strong potential and other one is weak exponentially decreasing potential

$$T \sim t_s t_w$$

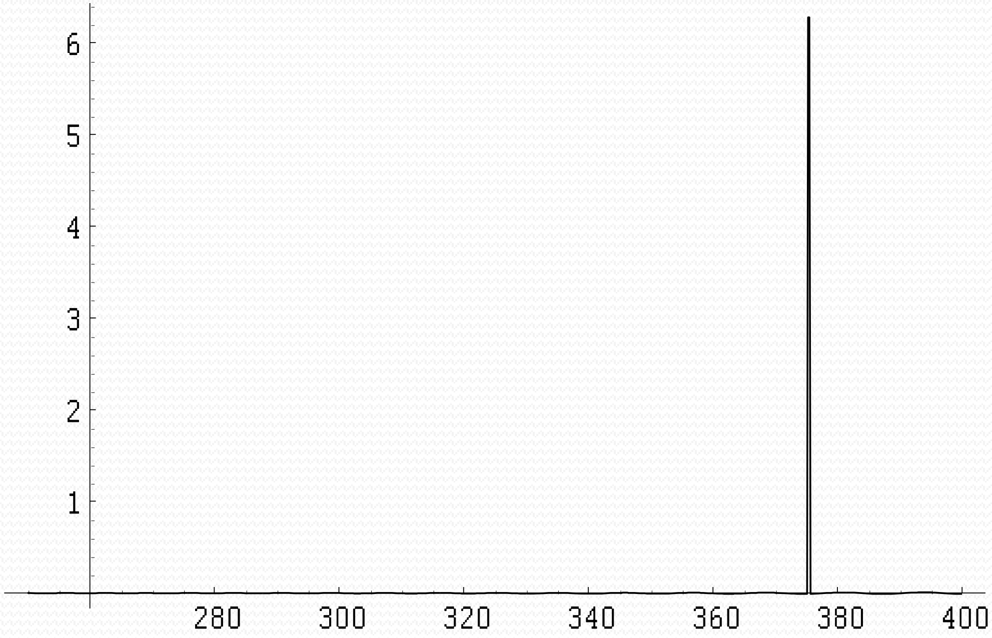
then

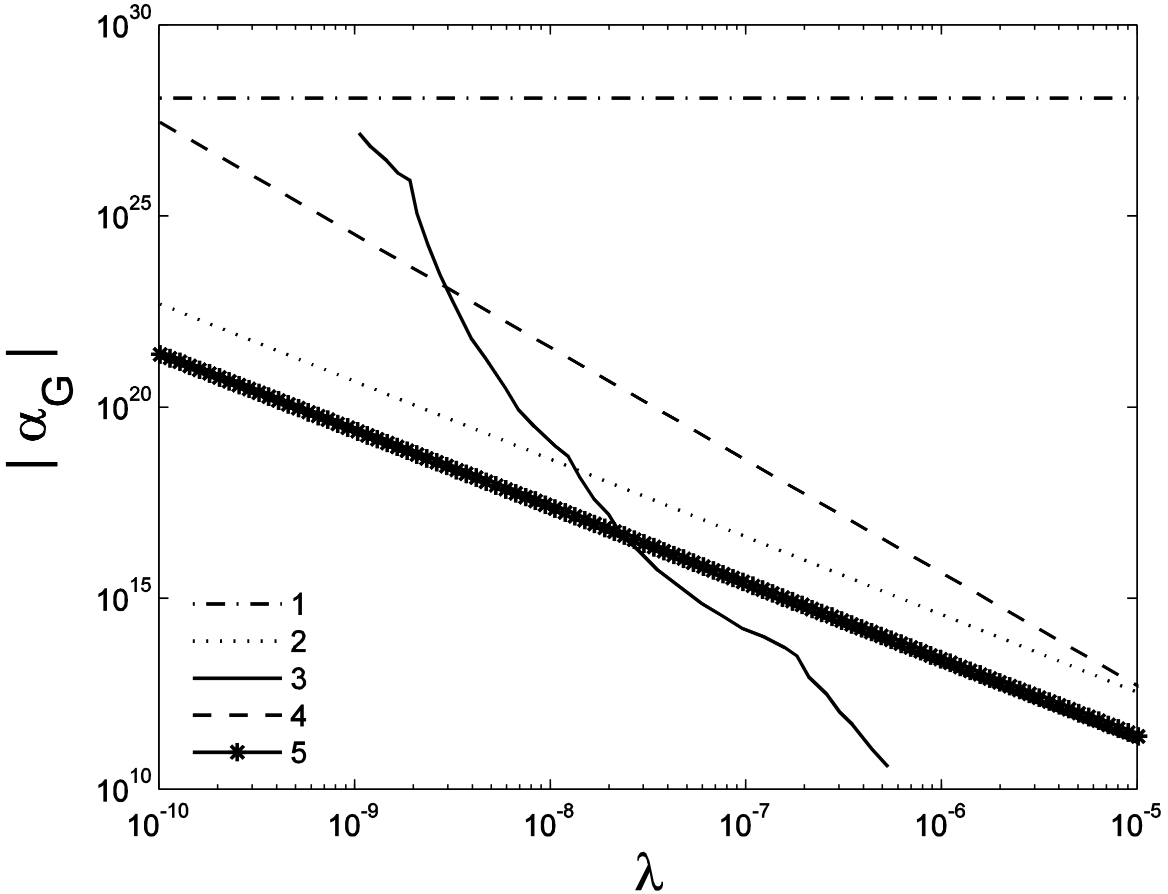
$$Arg(T) = Arg(t_s) + Arg(t_w) + \ldots$$

$$Arg(t_w) \sim \tan^{-1}(\pi\lambda_n / \lambda)$$

$$Arg(t_w) \sim \tan^{-1}\left(\frac{2(\pi\lambda_n / \lambda)}{(\pi\lambda_n / \lambda)^2 - 1}\right)$$

Phase shift for two slabs with L=10nm and λ=5nm as a function of neutron wavelength λ_n ($\mathring{A}$)





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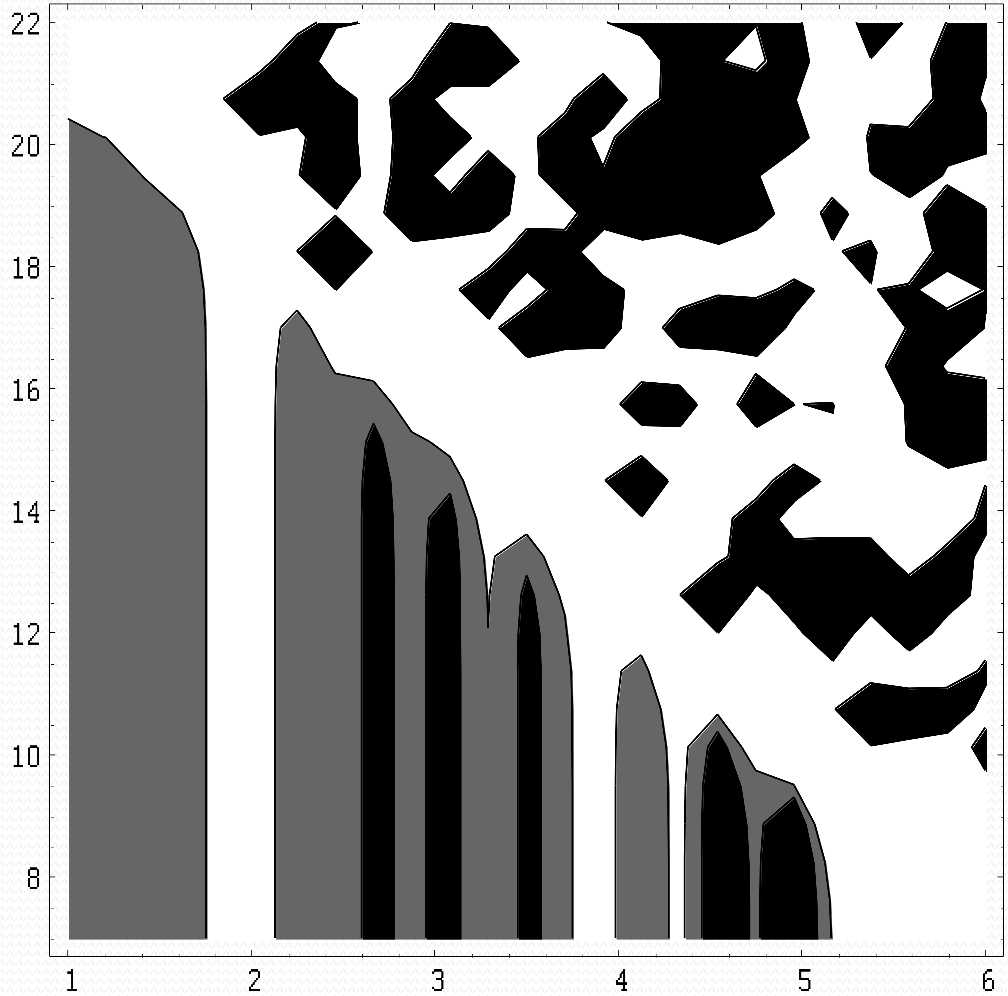
Experimental sensitivity to the phase at the level 10^{-3} *rad* ($\lambda_n = 300\,\mathring{A}$)



Summary

- Coherent interactions
- Low energy to “match” extremely small potentials
- Low energy to approach imaginary poles
- Resonance interactions