
Lepton Flavor Violation

Yasuhiro Okada (KEK/Sokendai)

March 6, 2008

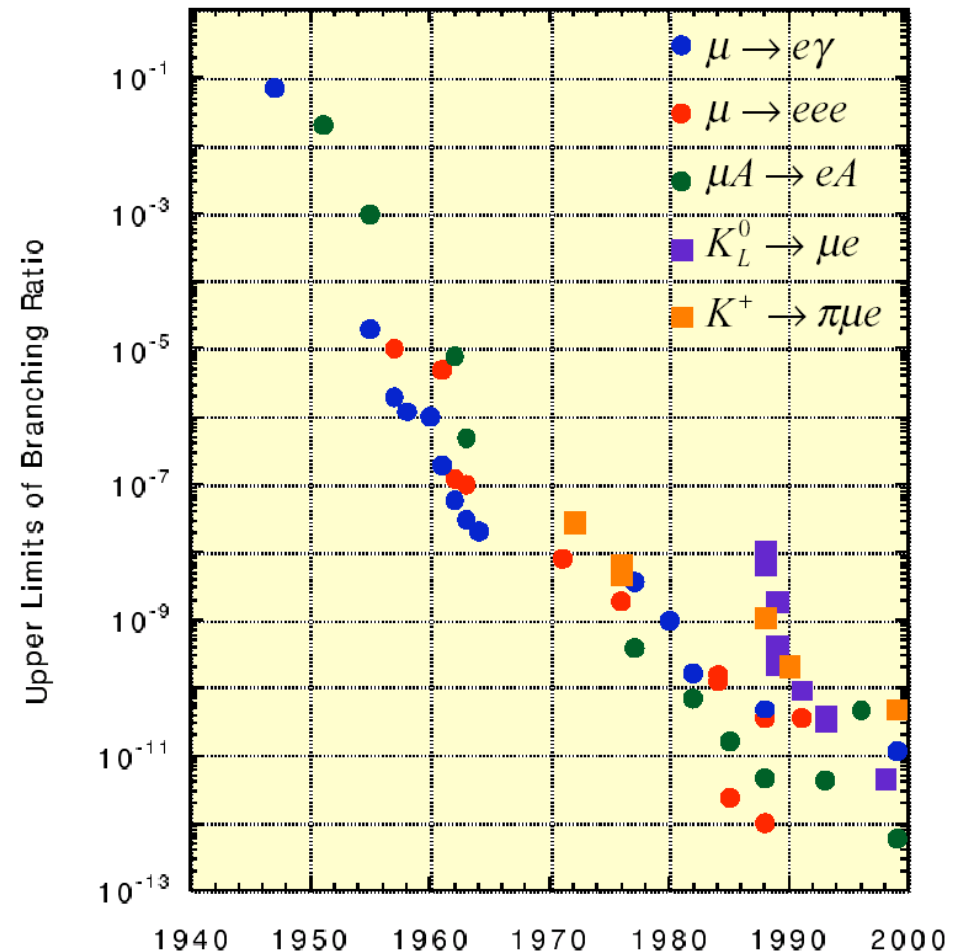
NP08, Mito, Japan

Lepton Flavor Violation in charged lepton processes

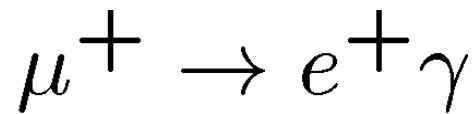
- Charged lepton LFV is a clear evidence of physics beyond the Standard Model.
- Neutrino mixing suggests existence of lepton flavor mixing. Large LFV in charged lepton processes is possible, if there is new physics at the TeV scale.
- There are various observable quantities in muon LFV processes which are useful to distinguish different models.

Lepton Flavor Violation

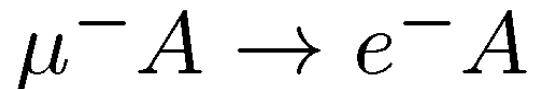
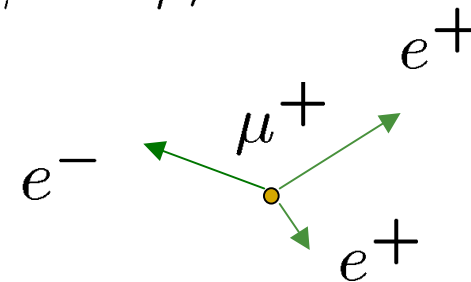
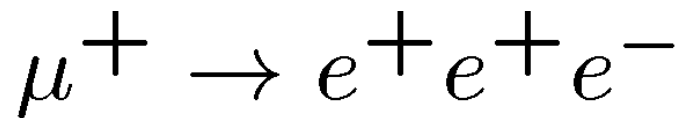
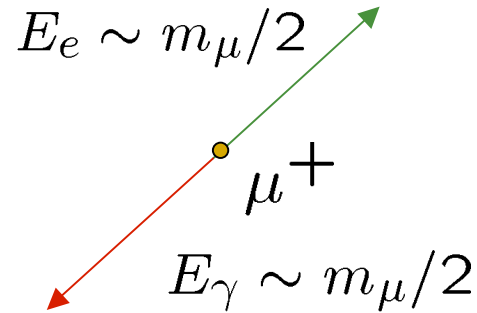
- No lepton flavor violation (LFV) in the Standard Model.
- LFV in charged lepton processes is negligibly small for a simple seesaw neutrino model.



Three muon LFV processes

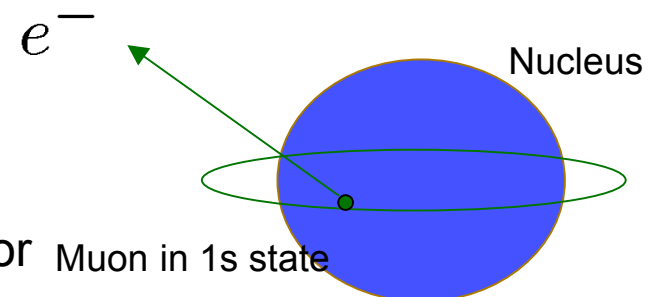


Back to back emission of a positron and a photon with an energy of a half of the muon mass.



$$E_e \sim m_\mu - E_{1s \text{ Binding}}$$

A monochromatic energy electron emission for the coherent mu-e transition. Muon in 1s state

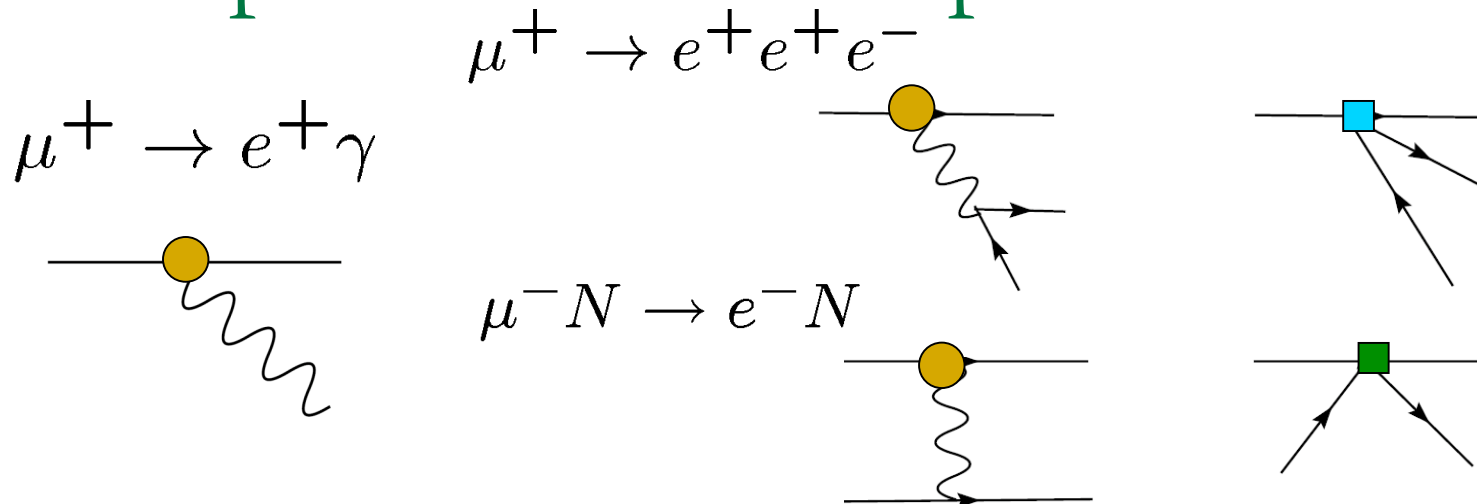


Experimental bounds

Process	Current	Near future
$\mu^+ \rightarrow e^+ \gamma$	1.2×10^{-11}	$\sim 10^{-13}$ (MEG)
$\mu^+ \rightarrow e^+ e^+ e^-$	1.0×10^{-12}	
$\mu^- N \rightarrow e^- N$ (Ti)	6.1×10^{-13}	
$\Delta L_f = 2$ $G_{Mu\bar{M}u}/G_F$	3×10^{-3}	

Current bounds on tau LFV processes ($\tau \rightarrow \mu \gamma, \tau \rightarrow 3 \mu, \tau \rightarrow \mu \eta$, etc.) are $0(10^{-8})$ - $0(10^{-7})$.

Comparison of three processes



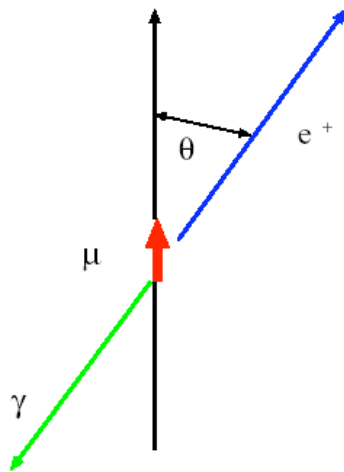
If the photon penguin process is dominated, there are simple relations among these branching ratios.

$$\begin{aligned}
 B(\mu \rightarrow 3e) &\sim 6.1 \times 10^{-3} B(\mu \rightarrow e\gamma) \\
 B(\mu T_i \rightarrow e T_i) &\sim 4.0 \times 10^{-3} B(\mu \rightarrow e\gamma) \\
 B(\mu A_l \rightarrow e A_l) &\sim 2.6 \times 10^{-3} B(\mu \rightarrow e\gamma)
 \end{aligned}$$

In many case of SUSY modes, this is true, but there is an important case in which these relations do not hold.

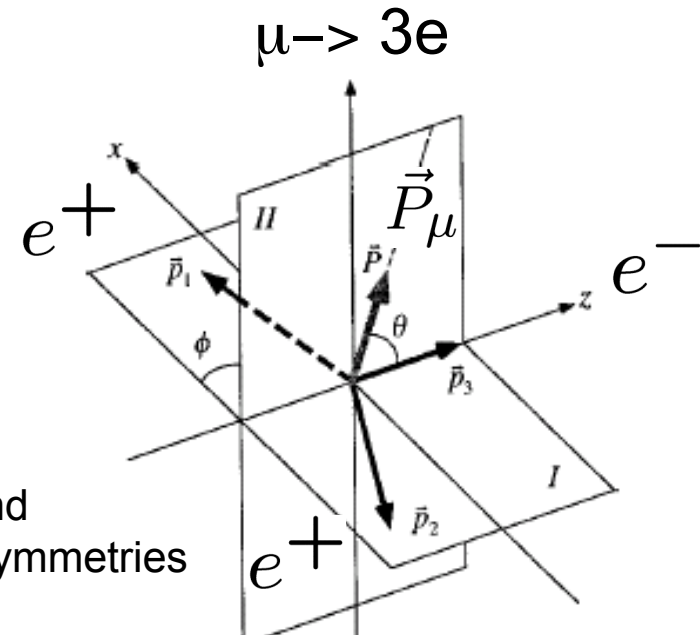
Muon polarization and LFV processes

- If the muon is polarized, we can define a P-odd asymmetry for $\mu \rightarrow e \gamma$ and T-odd and P-odd asymmetries for $\mu \rightarrow 3e$. These asymmetries are useful to distinguish different models.



$$\frac{dB(\mu^+ \rightarrow e^+ \gamma)}{d \cos \theta} \propto 1 + A_{\mu \rightarrow e \gamma} P_{\mu} \cos \theta$$

Example :
A = -1 for the SUSY seesaw model



Two P-odd and one T-odd asymmetries

$$A_{P_1} = \frac{N(P_z > 0) - N(P_z < 0)}{N(P_z > 0) + N(P_z < 0)}$$

$$A_{P_2} = \frac{N(P_x > 0) - N(P_x < 0)}{N(P_x > 0) + N(P_x < 0)}$$

$$A_T = \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)}$$

$\mu \rightarrow e \gamma$ and $\mu \rightarrow 3e$ asymmetries in SUSY models

P and T-odd asymmetries in minimal SUSY GUT models

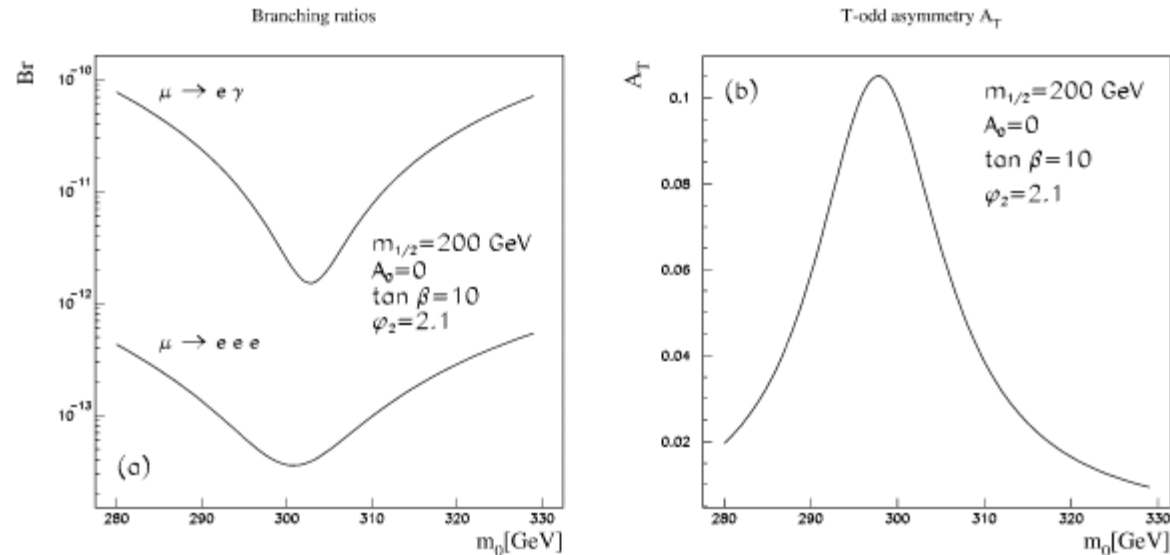
The T-odd asymmetry can be 10 % level for some parameter space of the SU(5) SUSY GUT and the SUSY seesaw model.

	SU (5)	SO (10)
$A_{\mu \rightarrow e \gamma}$	+100%	-100% - +100%
A_{P_1}	-30% - +40 %	$\simeq -A_{\mu \rightarrow e \gamma} / 10$
A_{P_2}	-20% - +20 %	$\simeq -A_{\mu \rightarrow e \gamma} / 6$
$ A_T $	$\lesssim 15\%$	$\lesssim 0.01\%$

Y.Okada,K.Okumura,and Y.Shimizu, 2000

T-odd asymmetry in the SUSY seesaw model

Information on lepton sector CP violation



J.Ellis,J.Hisano,S.Lola, and M.Raidal, 2001

Z dependence of mu-e conversion branching ratio

R.Kitano, M.Koike and Y.Okada. 2002

We have calculated the coherent mu-e conversion branching ratios in various nuclei for general LFV interactions to see:

- (1) which nucleus is the most sensitive to mu-e conversion searches,
- (2) whether we can distinguish various theoretical models by the Z dependence.

Relevant quark level interactions

$$\begin{aligned} \mathcal{L}_{\text{int}} = & -\frac{4G_F}{\sqrt{2}} (m_\mu A_R \bar{\mu} \sigma^{\mu\nu} P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu} \sigma^{\mu\nu} P_R e F_{\mu\nu} + \text{h.c.}) \quad \leftarrow \text{Dipole} \\ & -\frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \left[(g_{LS(q)} \bar{e} P_R \mu + g_{RS(q)} \bar{e} P_L \mu) \bar{q} q \quad \leftarrow \text{Scalar} \right. \\ & \quad \left. + (g_{LV(q)} \bar{e} \gamma^\mu P_L \mu + g_{RV(q)} \bar{e} \gamma^\mu P_R \mu) \bar{q} \gamma_\mu q \quad \leftarrow \text{Vector} \right. \\ & \quad \left. + \text{h.c.} \right], \end{aligned}$$

Calculation of μ -e conversion rate

The μ -e conversion rate is defined

$$B \equiv \frac{\omega(\mu N(A, Z) \rightarrow e N(A, Z))}{\omega(\mu N(A, Z) \rightarrow \text{capture})}$$

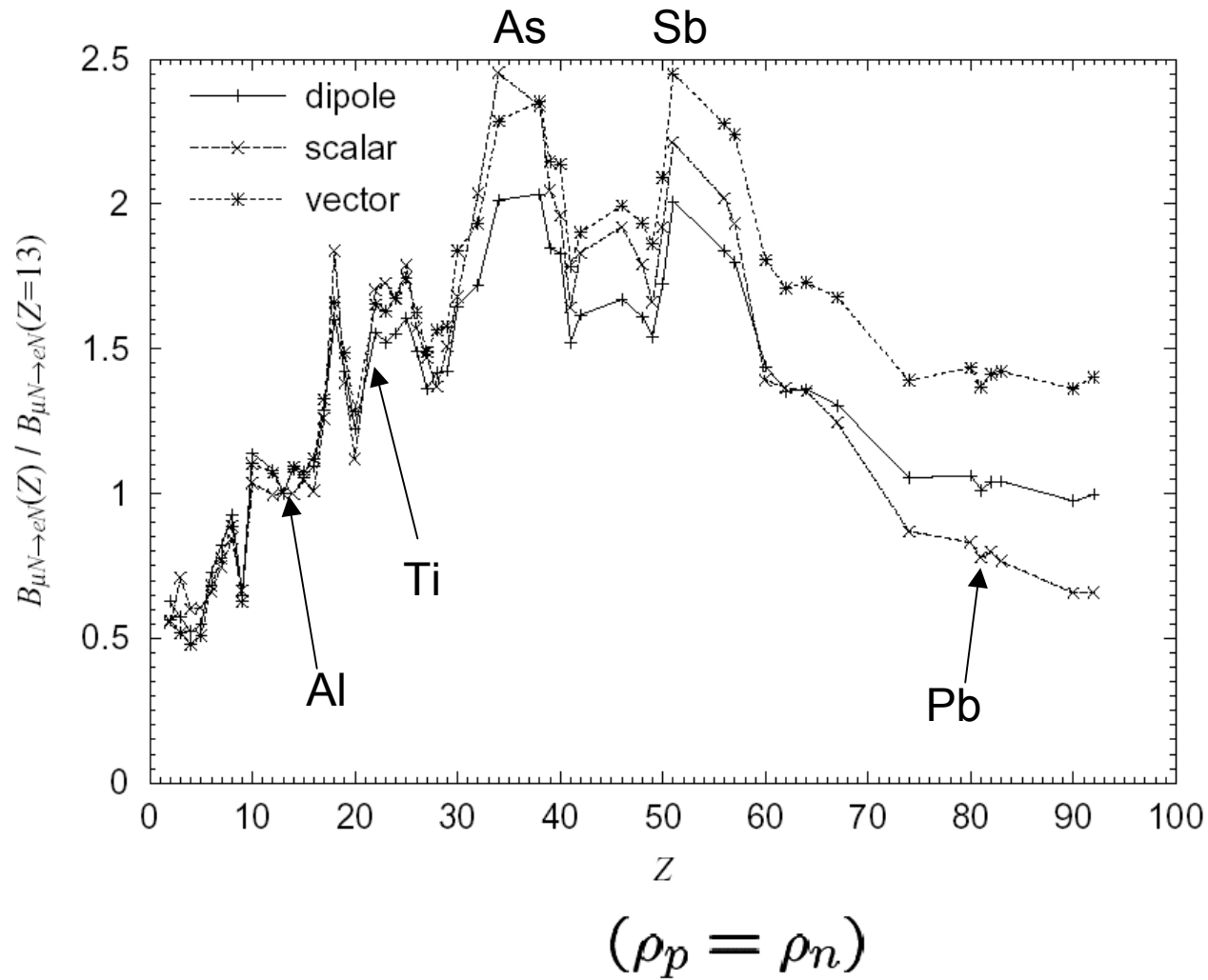
Schematically,

$$\begin{aligned} & \omega(\mu N(A, Z) \rightarrow N(A, Z)) \\ & \sim \left| \sum \int d^3x \bar{\psi}_e \Gamma \psi_\mu^{(1S)} \rho^{(p,n)} \right| \end{aligned}$$

This depends on

- (i) the matrix element of quark operators in a nucleon state,
- (ii) the proton and neutron distributions in a nucleus,
- (iii) the type of the lepton operator

mu-e conversion rate normalized at Al.



(1) The branching ratio is largest for the atomic number of $Z=30 - 60$.
“Form factor” effect for large nuclei.

(2) For light nuclei, Z dependences are similar for different operator forms.
Sizable difference of Z dependences for dipole, scalar and vector interactions. This is due to a relativistic effect of the muon wave function.

$$B(\mu Pb \rightarrow e Pb) / B(\mu Al \rightarrow e Al) = 1.0, \quad 0.77, \quad 1.4$$

dipole scalar vector

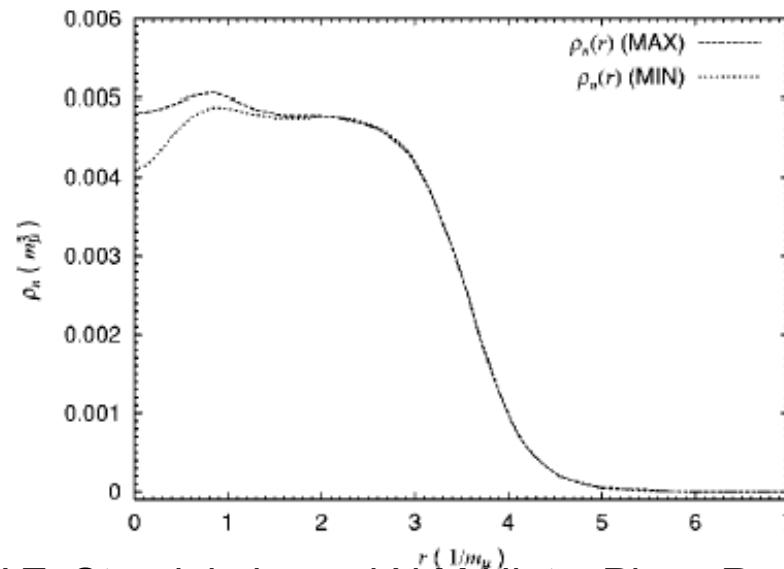
(3) Theoretical ambiguity depends on type of LFV operators.
Little uncertainty for the photonic operators.
(Charge distributions of nuclei are well known)

Example: uncertainty of overlapping integrals for Pb due to neutron distribution .

		Proton			Neutron	
		dipole	scalar	vector	scalar	vector
		D	$S^{(p)}$	$V^{(p)}$	$S^{(n)}$	$V^{(n)}$
$^{208}_{82}\text{Pb}$	Minimum	0.163	0.0493	0.0845	0.0675	0.119
	Maximum	0.163	0.0493	0.0845	0.0697	0.121

Ambiguity from neutron distribution is a few %.

Neutron distribution from proton scattering.



V.E. Staudubsky and N.M.Hintz, Phys. Rev C, 1994

Supersymmetry and LFV

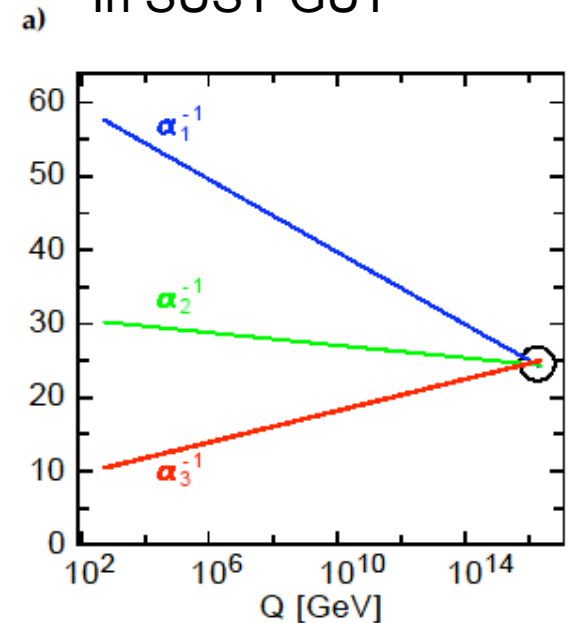
- Supersymmetry is a new type of symmetry connecting bosons and fermions.
- Gauge coupling unification is realized with SUSY particles.
- The LHC experiment can find squarks and gluon up to 2-3 TeV.

SM particles

quark lepton gluon W,Z, γ , H	}	Spin 1/2	}	squark (\tilde{q})
		Spin 0		slepton (\tilde{l})
	Spin 1	Spin 1/2	gluino (\tilde{g})	
	Spin 1	}	neutralino, chargino ($\tilde{\chi}$)	
Spin 0				

Super partners

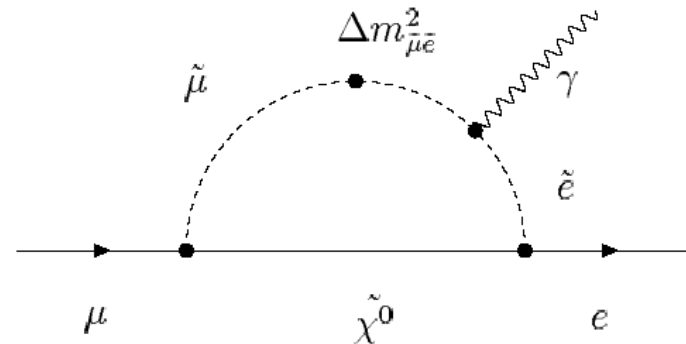
Coupling unification
In SUSY GUT



Slepton flavor mixing

In SUSY models, LFV processes are induced by the off-diagonal terms in the slepton mass matrixes

$$m_{\tilde{l}}^2 = \begin{pmatrix} m_{11}^2 & m_{12}^2 & m_{13}^2 \\ m_{21}^2 & m_{22}^2 & m_{23}^2 \\ m_{31}^2 & m_{32}^2 & m_{33}^2 \end{pmatrix}$$

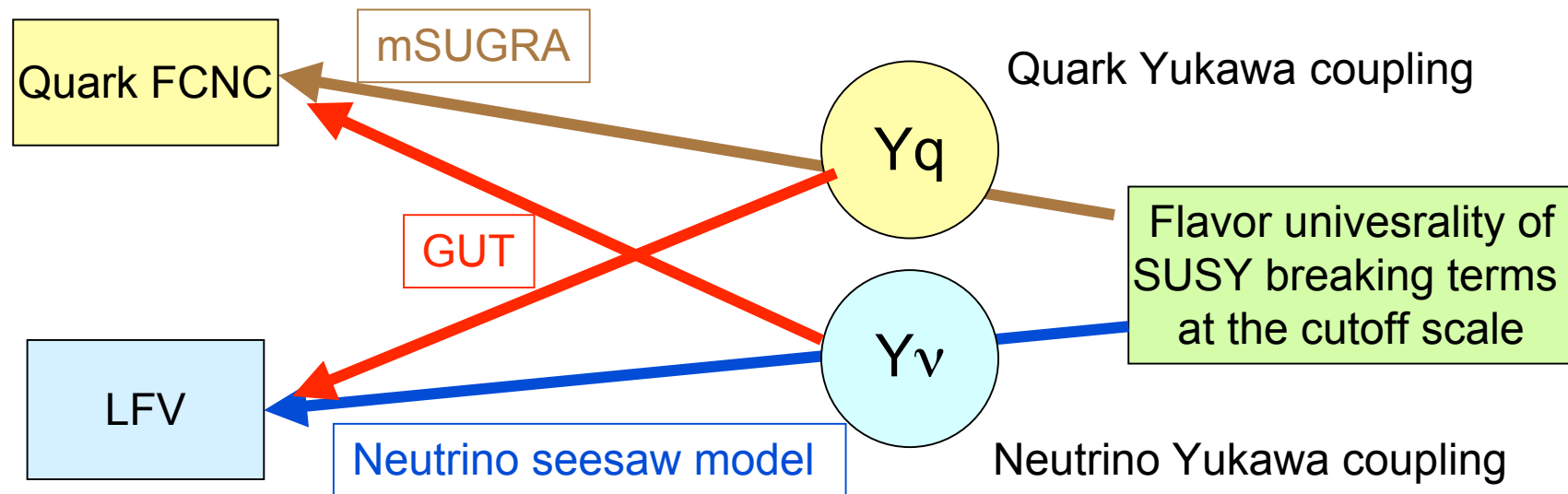


g-2: the diagonal term
EDM: complex phases
LFV: the off-diagonal term

Off-diagonal terms depend on how SUSY breaking is generated and what kinds of LFV interactions exist at the GUT scale.

SUSY GUT and SUSY Seesaw model

- Quark and neutrino Yukawa couplings are sources of squark and slepton flavor mixings.



L.J.Hall, V.Kostelecky, S.Raby, 1986; A.Masiero, F.Borzumati, 1986

$\mu \rightarrow e \gamma$ branching ratio (typical example)

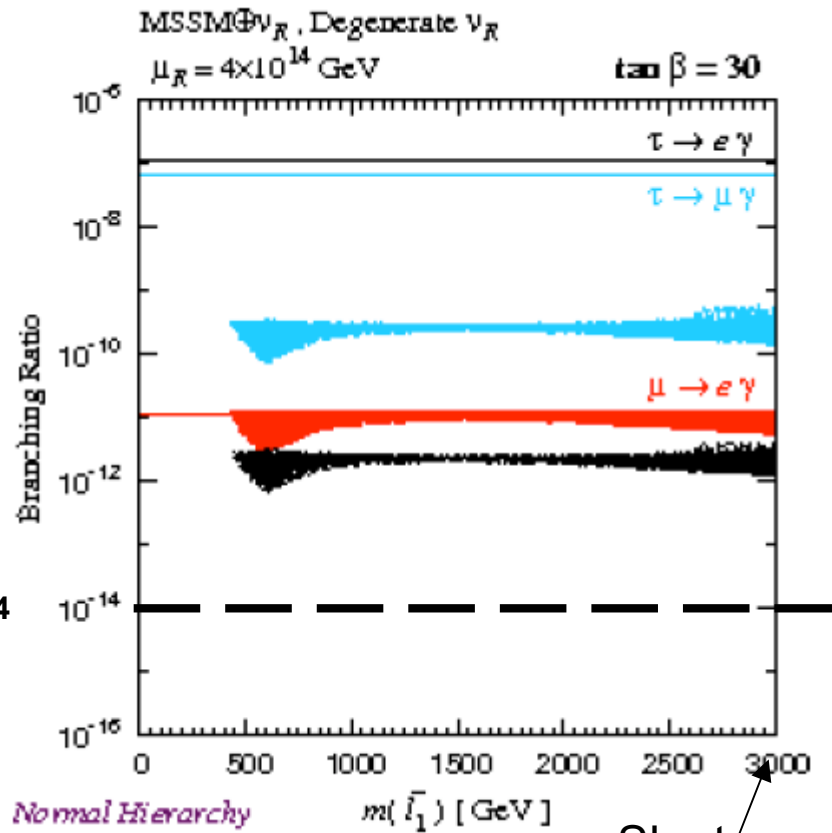
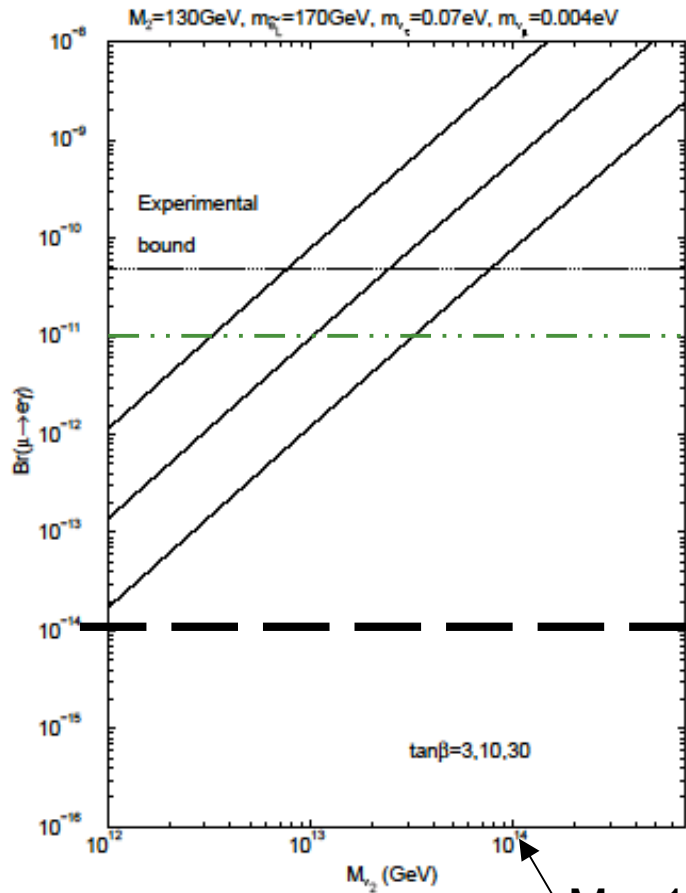
SUSY Seesaw model

$B(\mu \rightarrow e \gamma) \sim 10^{-14} \Leftrightarrow B(\mu N \rightarrow e N) \sim 10^{-16}$

slepton mass = 170 GeV

$M_R = 4 \times 10^{14}$ GeV

$\mu \rightarrow e \gamma$ in the MSSMRN with the MSW large angle solution

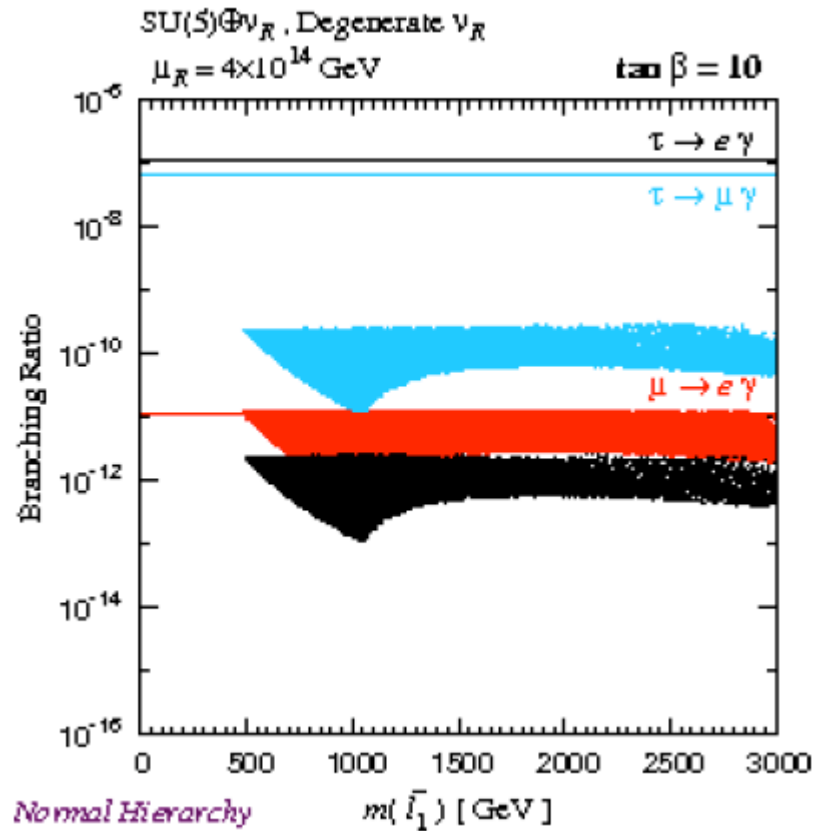


10^{-14}

Slepton mass = 3 TeV

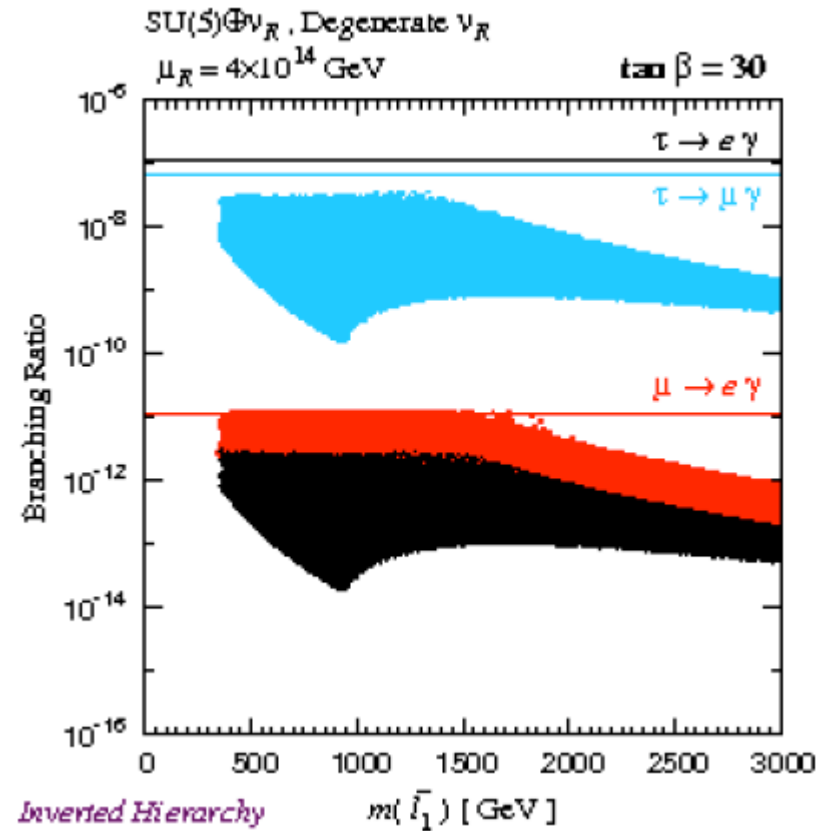
SU(5) SUSY GUT with right-handed neutrino

Neutrino mass normal hierarchy



$\mu \rightarrow e \gamma$ large

Neutrino mass inverse hierarchy



$\mu \rightarrow e \gamma, \tau \rightarrow \mu \gamma$ large

T.Goto, Y.Okada, T.Shindou, M.Tanaka, 2007

SUSY seesaw with a large $\tan \beta$

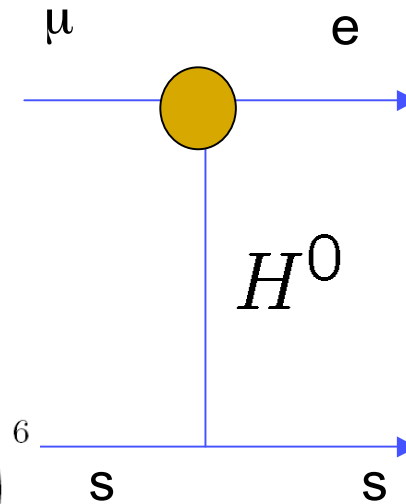
R.Kitano, M.Koike, S.Komine, and Y.Okada, 2003

SUSY loop diagrams can generate a LFV Higgs-boson coupling for large $\tan \beta$ cases. (K.Babu, C.Kolda, 2002)

The heavy Higgs-boson exchange provides a new contribution of a scalar type.

Higgs-exchange contribution

$$B(\mu Al \rightarrow e Al)_{H^0} \sim O(10^{-13}) \cdot \left(\frac{200 \text{ GeV}}{m_{H^0}} \right)^4 \cdot \left(\frac{\tan \beta}{60} \right)^6$$



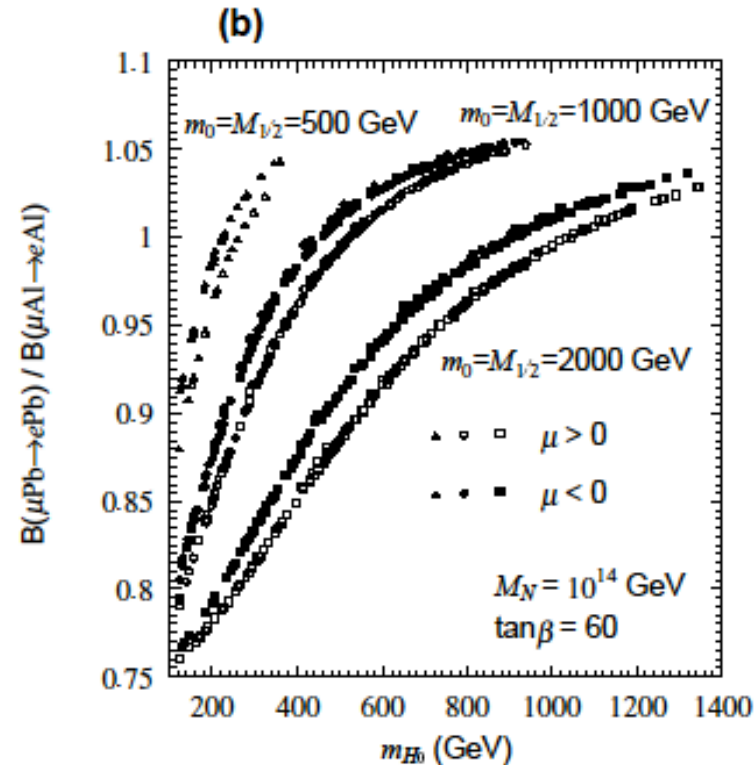
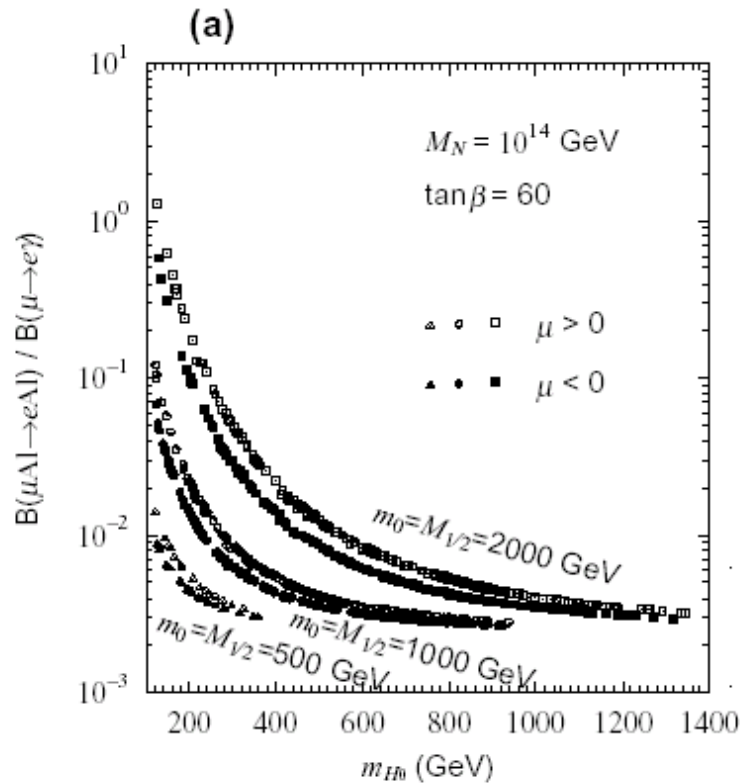
Photon-exchange contribution

$$B(\mu Al \rightarrow e Al)_{\gamma} \sim O(10^{-13}) \cdot \left(\frac{1000 \text{ GeV}}{M_S} \right)^4 \cdot \left(\frac{\tan \beta}{60} \right)^2$$

Ratio of the branching ratios and Z-dependence of mu-e conversion rates

$$B(\mu Al \rightarrow eAl) / B(\mu \rightarrow e\gamma)$$

$$B(\mu Pb \rightarrow ePb) / B(\mu Al \rightarrow eAl)$$



mu-e conversion is enhanced.

Z-dependence indicates the scalar exchange contribution.

Neutrino mass generation at TeV scale and LFV

- Although the simple seesaw or Dirac neutrino model predicts too small branching ratios for the charged lepton LFV, other models of neutrino mass generation can induce observable effects if there is a new source of lepton flavor mixing at TeV scale,

Examples

SUSY seesaw model (F.Borzumati and A.Masiero 1986)

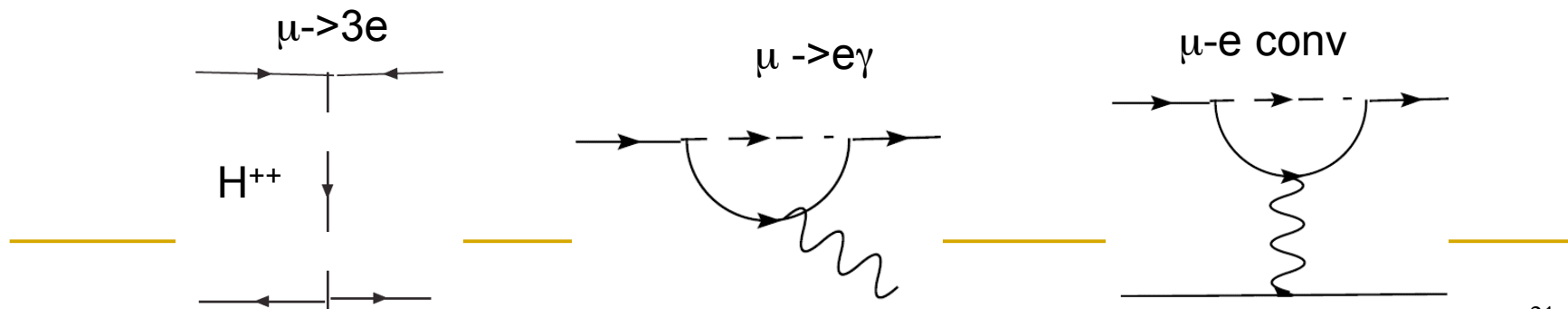
Generalized Zee model (K.Hasagawa, C.S.Lim, K.Ogure, 2003)

Neutrino mass from the warped extra dimension (R.Kitano,2000)

R-parity violating SUSY model (A.de Gouvea,S.Lola,K.Tobe,2001)

Triplet Higgs model (E.J.Chun, K.Y.Lee,S.C.Park; N.Kakizaki,Y.Ogura, F.Shima, 2003)

Left-right symmetric model (V.Cirigliano, A.Kurylov, M.J.Ramsey-Musolf, P.Vogel, 2004)

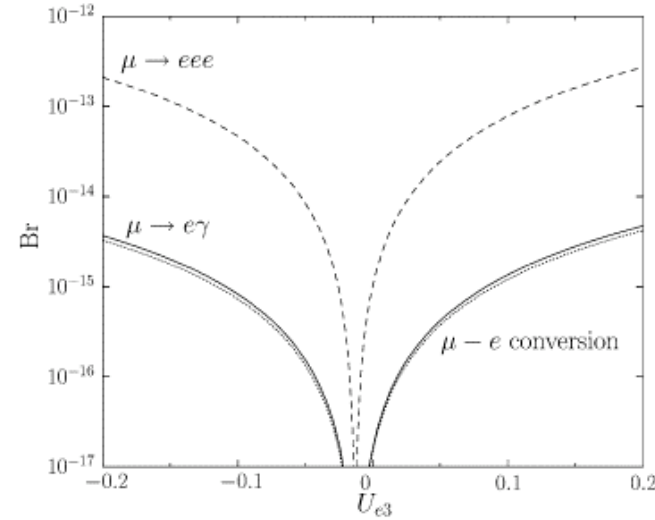
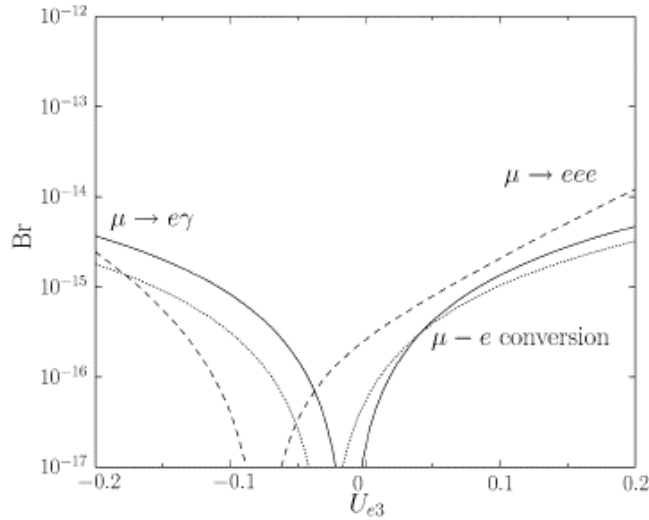


Triplet Higgs model

N.Kakizaki, Y.Ogura, F.Shima

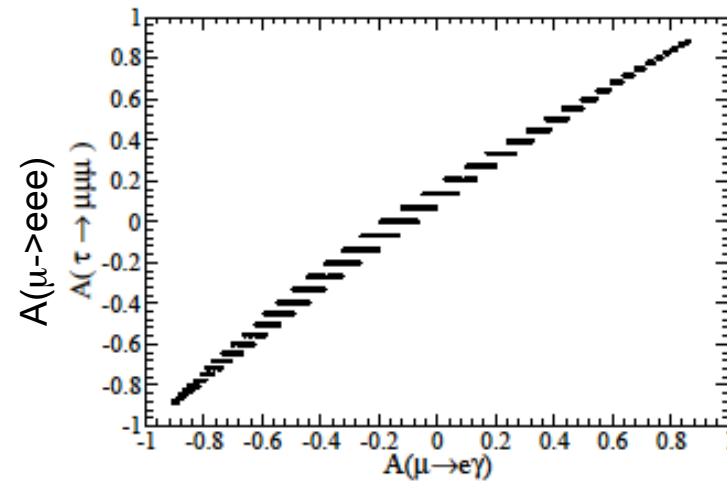
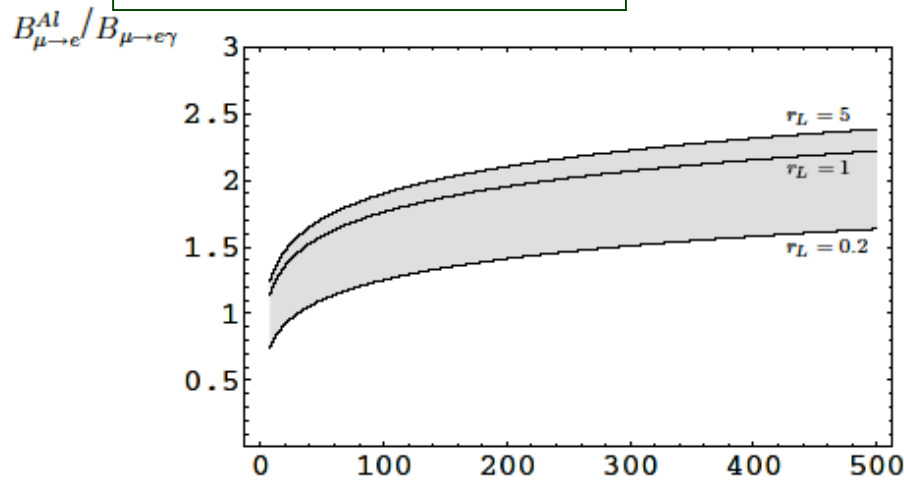
Hierarchical case

Degenerate case



LR symmetric model

$\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ asymmetries



V.Cirigliano, A.Kurylov, M.J.Ramsey-Musolf, P.Vogel, M_{W_2}/M_{W_1}

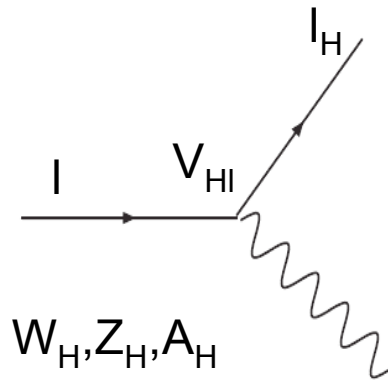
A.Akeroyd, M.Aoki and Y.Okada, 2006 22

Little Higgs model with T parity

Little Higgs models are a new type of composite Higgs models. New particles (heavy gauge bosons, a heavy top partner) are introduced to cancel the quadratic divergence of the Higgs mass at one loop level.

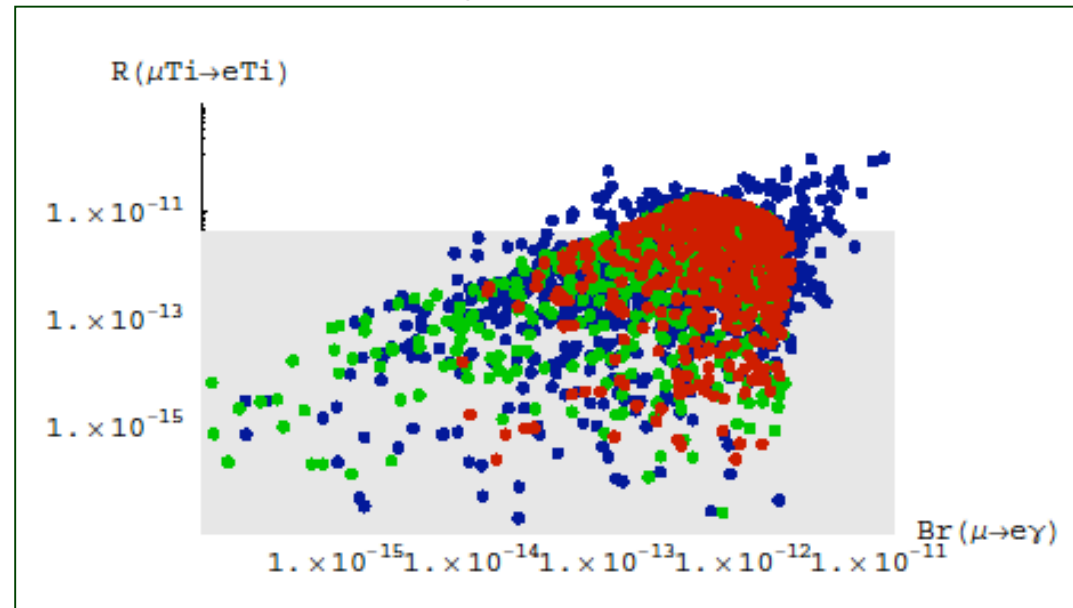
The mass of these particles are around 1 TeV if the model is extended with “T parity”.

T-odd SU(2) doublet mirror fermions
=> A new flavor mixing matrix independent of the MNS matrix.



W_H, Z_H, A_H

Blanke,Buras,Duling,Poschenrieder,Tarantino,2007



$0.01 < B(\mu N \rightarrow e N) / B(\mu \rightarrow e \gamma) < 100$, $0.4 < B(\mu \rightarrow 3e) / B(\mu \rightarrow e \gamma) < 2.5$.
The ratios can be quite different from SUSY case.

Comparison of three muon processes in various new physics models

SUSY GUT/Seesaw	$B(\mu \rightarrow e \gamma) \gg B(\mu \rightarrow 3e) \sim B(\mu N \rightarrow e N)$ Various asymmetries in polarized μ decays
SUSY with large $\tan \beta$	$B(\mu \rightarrow e \gamma) > B(\mu N \rightarrow e N) > B(\mu \rightarrow 3e)$ Z-dependence in μ -e conv. branching ratio
Triplet Higgs for neutrino	$B(\mu \rightarrow 3e) \gg B(\mu \rightarrow e \gamma) \sim B(\mu N \rightarrow e N)$ or $B(\mu \rightarrow 3e) \sim B(\mu \rightarrow e \gamma) \sim B(\mu N \rightarrow e N)$
RL model	$B(\mu \rightarrow 3e) \gg B(\mu \rightarrow e \gamma) \sim B(\mu N \rightarrow e N)$ in generic cases. Asymmetry in $\mu \rightarrow 3e$
Little Higgs model with T parity	$0.01 < B(\mu N \rightarrow e N)/B(\mu \rightarrow e \gamma) < 100$ $B(\mu \rightarrow 3e) \sim B(\mu \rightarrow e \gamma)$

Summary

- Muon LFV experiments provide various opportunities to search for new physics effects.
- Large effects are expected in well-motivated models of SUSY for LFV processes.
- If there are new particles at the TeV region related to the lepton flavor mixing, the neutrino oscillation may have some connection to the charged lepton LFV processes.