Theory of Muon $g - 2$

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I. Introduction/Overview
II. Standard Model contributions
  – hadronic contributions
III. SUSY contributions
IV. Summary

Partly based on **K. Hagiwara, A.D. Martin, DN and T. Teubner (HMNT)**,
Phys. Lett. **B557** (2003) 69; *Phys. Rev.* **D69** (2004) 093003;
Phys. Lett. **B649** (2007) 173.

Muon $g-2$ — Introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad \left(\vec{s} = \frac{1}{2}\vec{\sigma} \ \ \text{(spin)}, \qquad g = 2 + 2F_2(0)\right)$$

where

$$\overline{u}(p+q)\Gamma^{\mu}u(p) = \overline{u}(p+q)\left(\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)

Today, still important, since...

- ★ One of the **most precisely measured** quantities

$$\boxed{a_{\mu}^{\text{exp}} = 11\ 659\ 208.0(6.3) \times 10^{-10}} \quad [\mathbf{0.5ppm}] \qquad \text{(Bennett } et\ al\text{)}$$

- ★ **Extremely useful** in **probing/constraining physics beyond the SM**

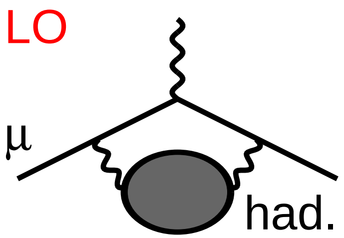
Recent Ups and Downs of Muon $g-2$

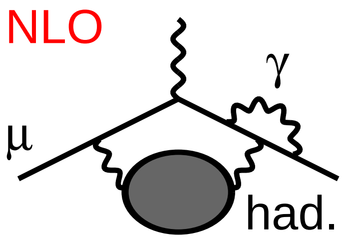
		EXP − TH
Feb '01	new exp. result (BNL)	**2.6 σ**
Nov '01	The 'famous' **l-by-l sign error found**	2.6 σ → **1.6 σ**
	(Knecht & Nyffeler)	
Dec '01	new $e^+e^- \longrightarrow \pi^+\pi^-$ data (CMD-2)	
July '02	new exp. result (BNL)	1.6 σ → **2.6 σ**
Aug '02 —	new eval. of the LO had. contribution	2.6 σ → **3.0 σ** (DEHZ, e^+e^-)
	using the new CMD-2 data	**3.3 σ** (HMNT, e^+e^-)
	(DEHZ, HMNT, Jegerlehner)	**(0.9 σ)** (DEHZ, τ)
Aug '03	**error found in the CMD-2 data analysis**	3.3 σ → **2.4 σ**
Dec '03	new eval. of the **l-by-l** contribution	2.4 σ → **2.0 σ**
	(Melnikov & Vainshtein)	
Jan '04	**new exp result (BNL)**	2.0 σ → **2.9 σ**
Feb '04	improved **QED** calculation (Kinoshita & Nio)	2.9 σ → **2.7 σ**
July '04	new F_π data from KLOE	
June '05	new $e^+e^- \to \pi^+\pi^-$ data from SND	
Feb '06	**final report** from BNL exp. (Bennett et al)	
May '06	error found in the SND analysis	
Oct '06	new $e^+e^- \to \pi^+\pi^-$ data from CMD-2	
Nov '06 —	updated analysis of the LO had contrib.	**3.4 σ** (HMNT)
Nov '07	**possible solution to e^+e^- vs τ puzzle**	(Benayoun et al)

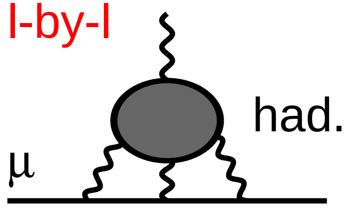
Standard Model Prediction for Muon $g-2$

QED contribution	11 658 471.809 (0.016) $\times10^{-10}$	Kinoshita & Nio
EW contrib.	15.4 (0.2) $\times10^{-10}$	Czarnecki et al
Hadronic contrib.		
LO hadronic	**689.4 (4.5)** $\times10^{-10}$	HMNT
NLO hadronic	$-$9.8 (0.1) $\times10^{-10}$	HMNT
light-by-light	13.6 (2.5) $\times10^{-10}$	Melnikov & Vainshtein
Theory TOTAL	**11 659 180.4 (5.1)** $\times10^{-10}$	
Experiment	**11 659 208.0 (6.3)** $\times10^{-10}$	world avg (Bennett et al (2006))
Exp $-$ Theory	**27.6 (8.1)** $\times10^{-10}$	**3.4 σ discrepancy**

n.b.: hadronic contributions:







The QED contribution to a_μ

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+\ 0.765857410\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+\ 24.05050964\ (43)\ (\alpha/\pi)^3$

Barbieri, Laporta, Remiddi, … , Czarnecki, Skrzypek, MP '04

$+\ 130.992\ (8)\ (\alpha/\pi)^4$ [See Nio's talk]

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04 & '05

$+\ 663\ (20)\ (\alpha/\pi)^5$ [See Nio's talk]

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.

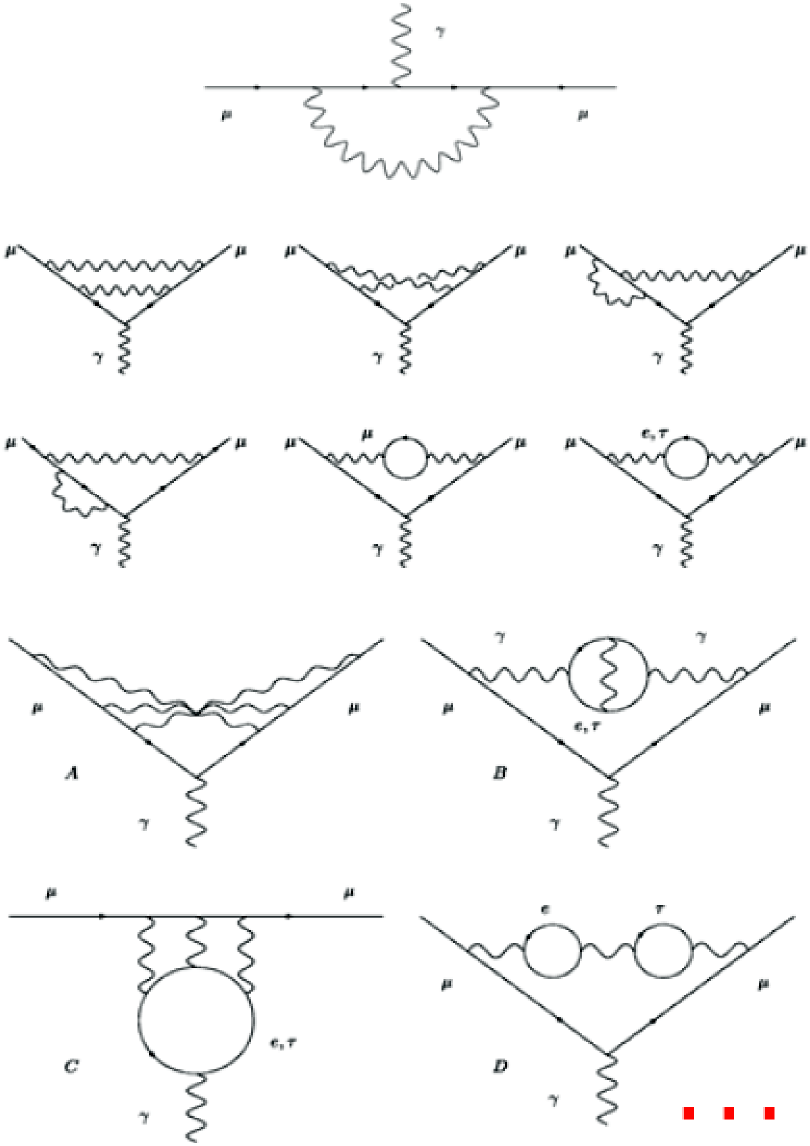
Aoyama-Hayakawa-Kinoshita-Nio, . . .

Adding up, I get:

$a_\mu^{QED} = 116584718.09\ (0.14)\ (0.08) \times 10^{-11}$

mainly from 5-loop unc ↲ ↳ from new $\delta\alpha$

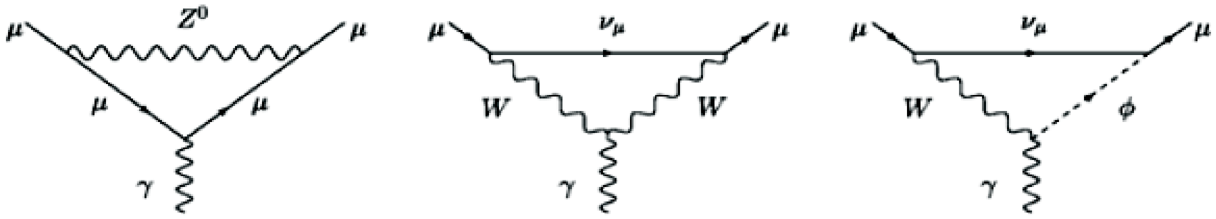
with $\alpha = 1/137.035999709\ (96)\ [0.7\text{ ppb}]$



Passera, talk at Tau06

The Electroweak contribution to a_μ

- **One-Loop Term:**



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda.

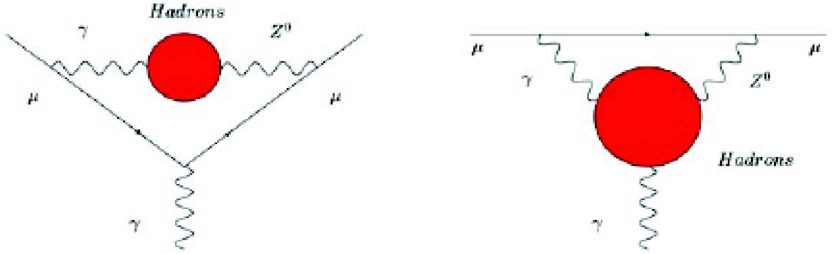
- **One-Loop plus Higher-Order Terms:**

$a_\mu^{\mathrm{EW}} = 154\ (2)\ (1) \times 10^{-11}$

Kukhto et al. '92; Czarnecki, Krause & Marciano '95; Knecht, Peris, Perrottet & de Rafael '02; Czarnecki, Marciano & Vainshtein '02; Degrassi & Giudice '98; Heinemeyer, Stockinger & Weiglein '04; Gribouk & Czarnecki '05; Vainshtein '03.

Higgs mass, M_top error, three-loop nonleading logs

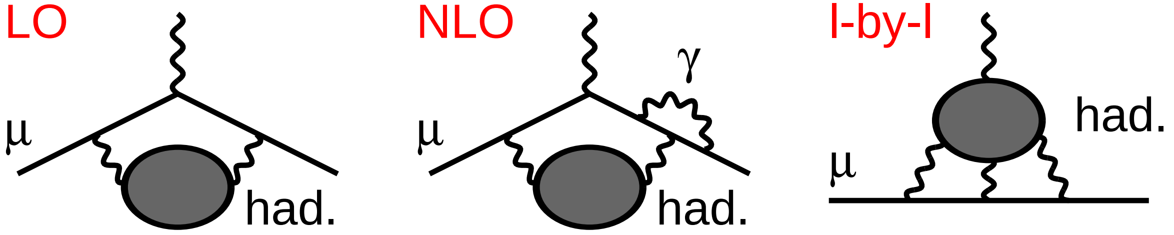
Hadronic loop uncertainties:



Passera, talk at Tau06

Hadronic contributions

$$\boxed{a_\mu^{\text{had}} = a_\mu^{\text{had, LO}} + a_\mu^{\text{had, NLO}} + a_\mu^{\text{l-by-l}}}$$



LO and NLO: calculable from exp. data

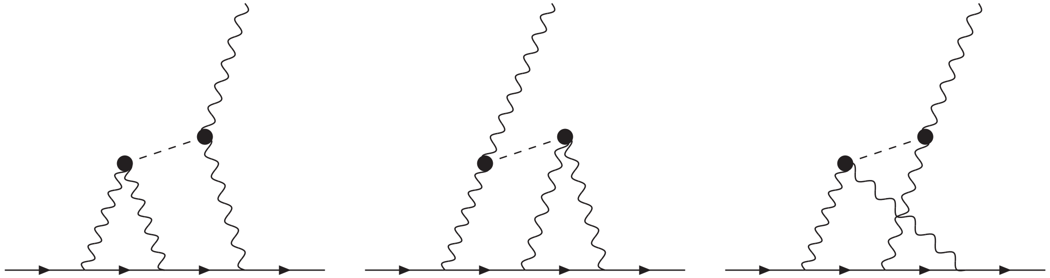
l-by-l: NOT calculable from exp. data, have to rely on model to some extent (model on pion form factor, large N_c expansion, ...)

There are some attempts to calculate them using lattice (Blum, Hayakawa-Blum-Izubuchi-Yamada, Aubin-Blum, . . .), but still suffering from large systematic uncertainties.

Light-by-light contribution

Modern procedure to compute the light-by-light contribution: (Melnikov and Vainshtein, ...)

1. First, use the large N_c expansion to find that the leading contribution is the pion pole contribution.

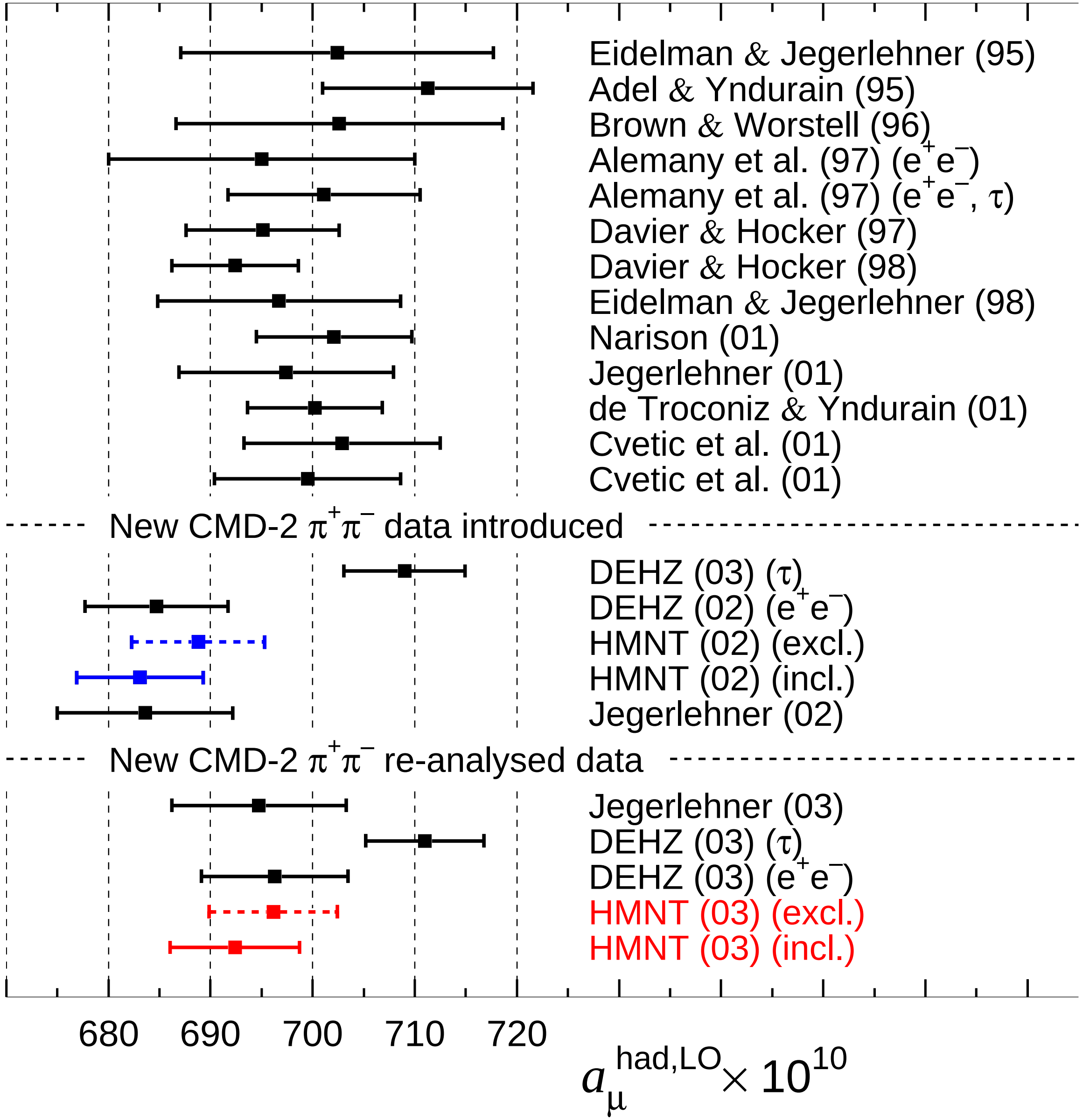


2. Choose the momentum-dependence of the $\pi\gamma\gamma$ coupling (form factor) in such a way that it is consistent with a constraint from QCD (OPE) at the momentum region $q_1^2 \sim q_2^2 \gg q_3^2$. Integrate over the loop momenta.

3. Repeat the above also for $\eta, \eta', a_1, \ldots$. Basically that's all for the LO in $1/N_c$.

As for NLO in $1/N_c$, there is more model dependence concerning which diagram is important.

Recent Evaluations of $a_\mu^{\text{had,LO}}$



✓ e^+e^--based evaluations
— convergent

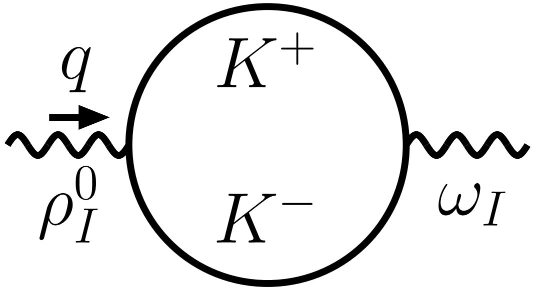
× Diff. between e^+e^--based and τ-based evaluations
— must be explained!

Solution to e^+e^- vs τ puzzle? (1)

Recently, a possible solution to the e^+e^- vs τ puzzle was proposed by Benayoun et al (arXiv:0711.4482).

Key observation: **Dynamical (*i.e.* s-dependent) mixing**:

The mixing among ρ_I^0, ω_I and ϕ_I (where I stands for "ideal" (i.e. $\phi_I \sim s\bar{s}$, $\omega_I \sim (u\bar{u} + d\bar{d})$, ...)) is naturally generated e.g. from 1-loop diagrams like



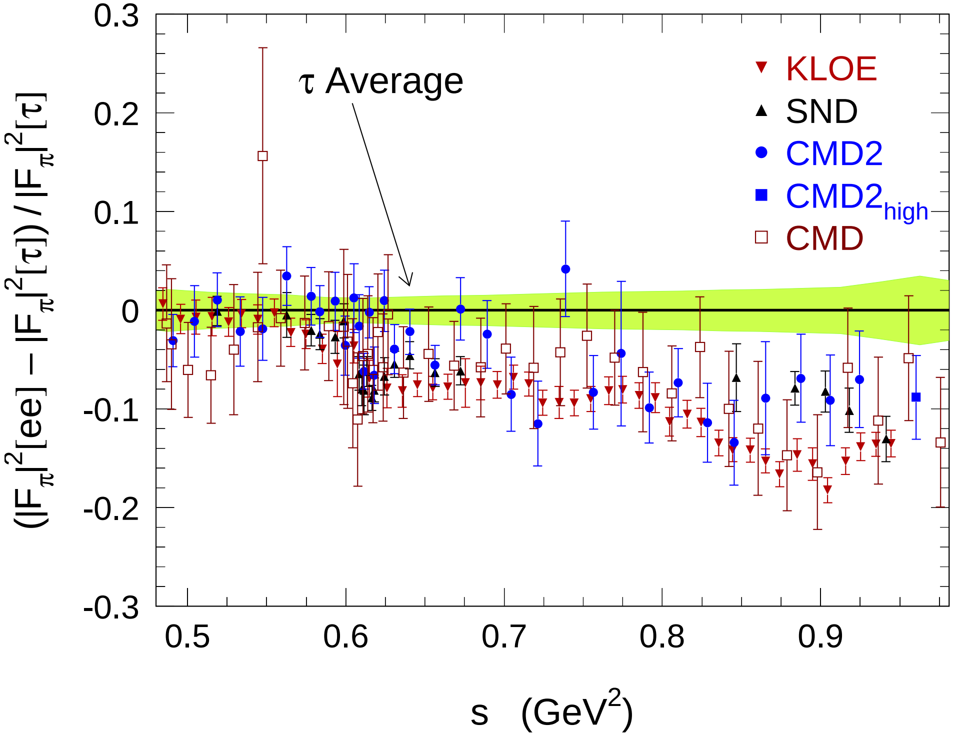
which depends on $s (\equiv q^2)$. Hence the mixing matrix is also s-dependent:

$$\begin{pmatrix} \rho^0 \\ \omega \\ \phi \end{pmatrix} = R(s) \begin{pmatrix} \rho_I^0 \\ \omega_I \\ \phi_I \end{pmatrix},$$

(The matrix $R(s)$ is an important input when converting the information from τ decays into e^+e^-. Very schematically, "$\rho^\pm \to \rho_I^0 \to \rho^0$".)

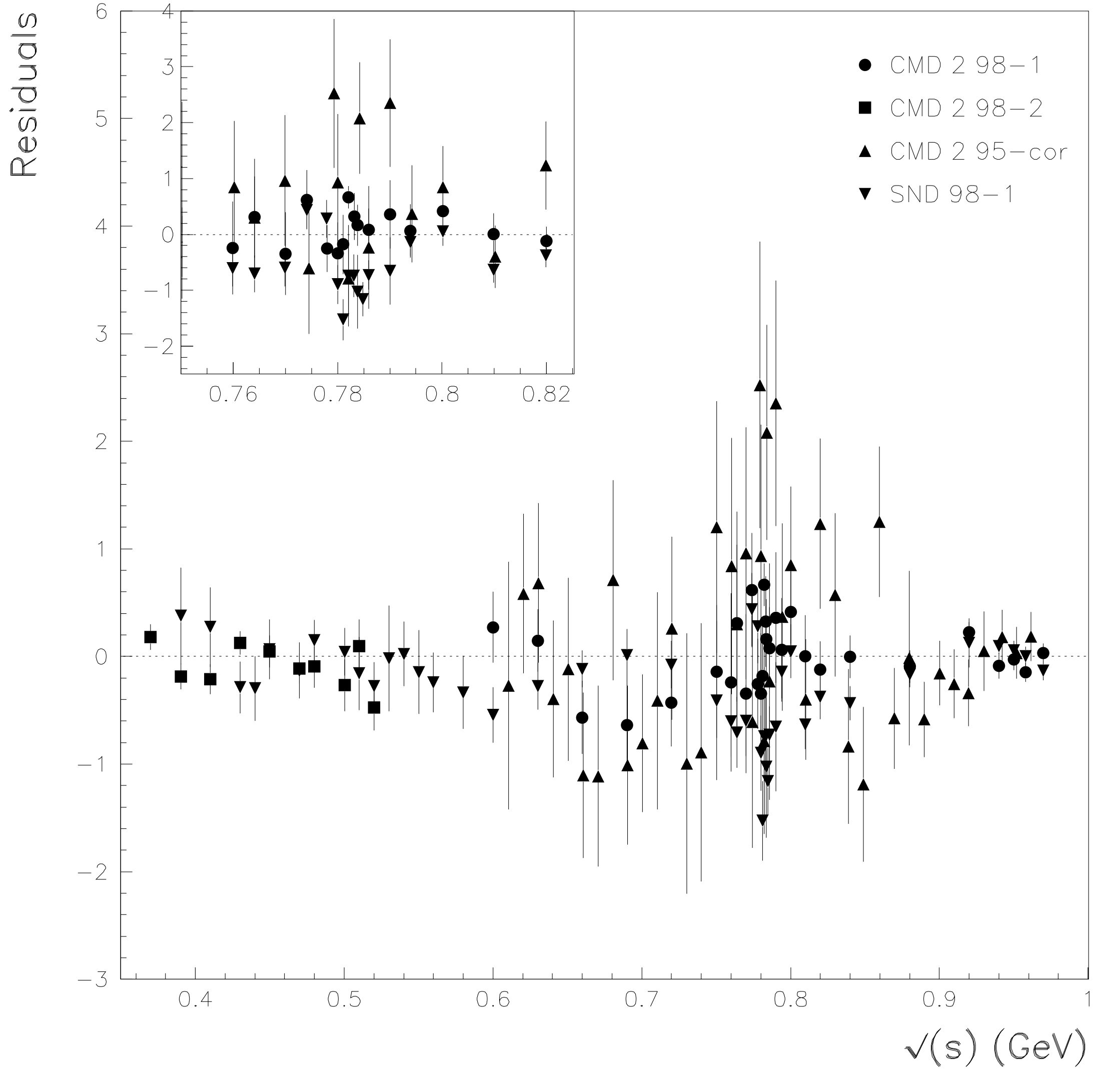
Solution to e^+e^- vs τ puzzle? (2)

Before Beyanoun et al:



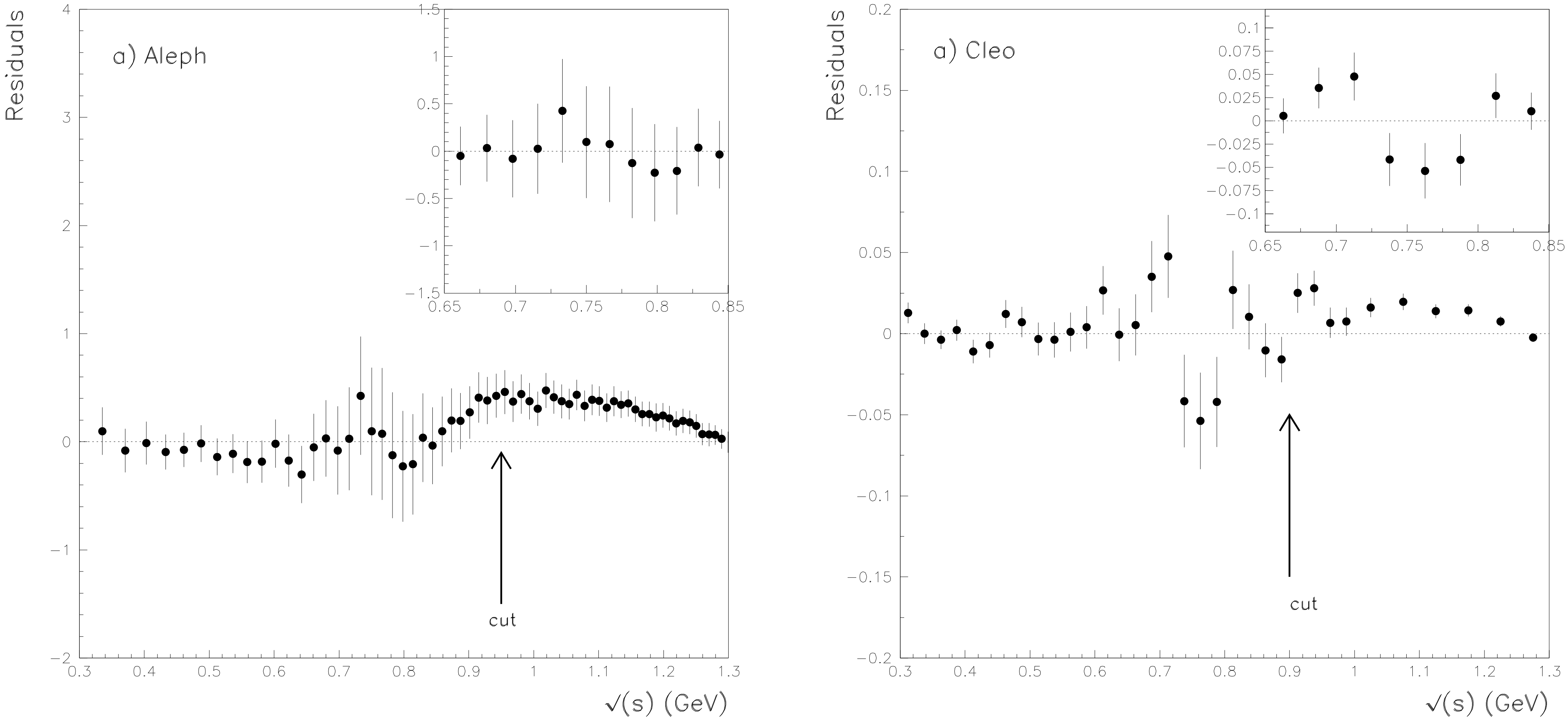
(from M. Davier, hep-ph/0701163)

After Benayoun et al:



(from Benayoun et al, arXiv:0711.4482)

Solution to e^+e^- vs τ puzzle? (3) — τ data vs fit

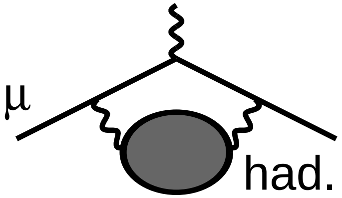


(figs taken from Benayoun et al, arXiv:0711.4482)

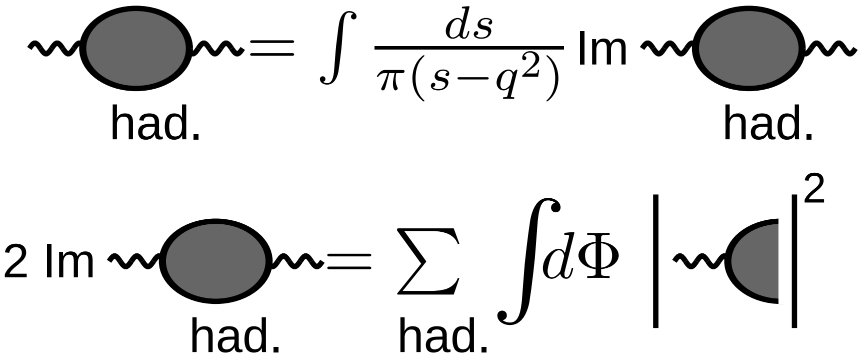
No consistency problem between the e^+e^- and τ data any longer

Evaluating $a_\mu^{\text{had,LO}}$

The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.

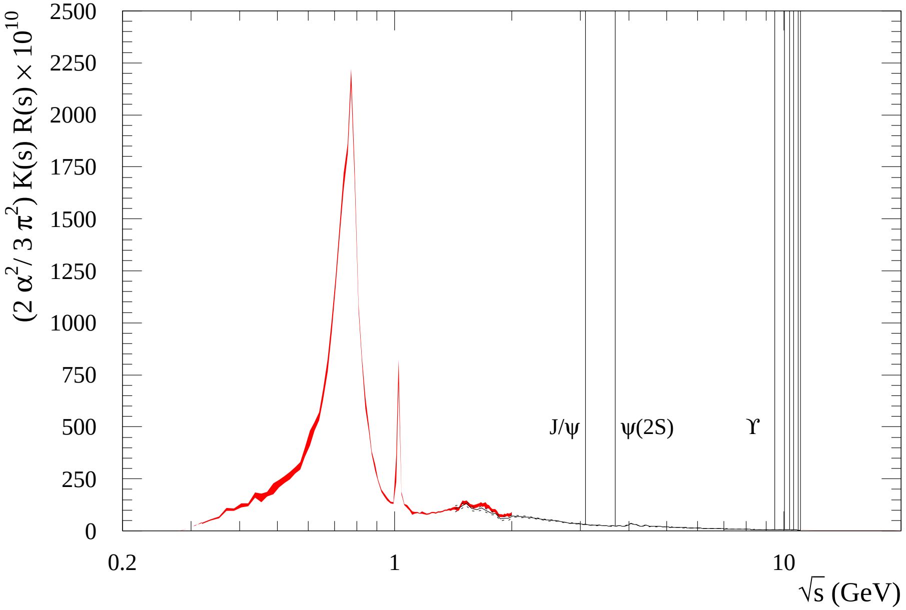


$$\boxed{a_\mu^{\text{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\text{th}}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\text{had}}(s)}$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

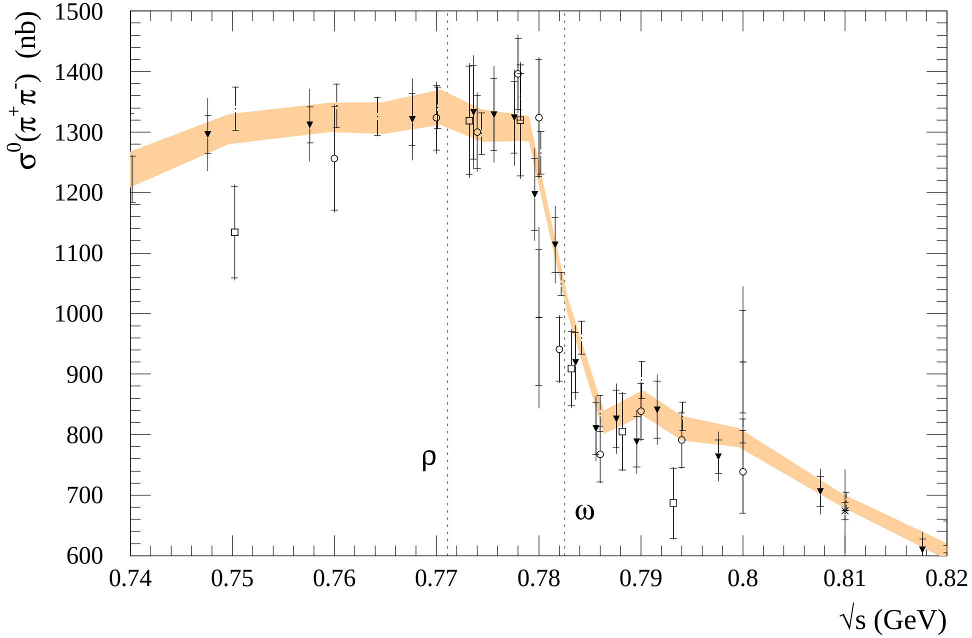
- We have to rely on exp. data for $\sigma_{\text{had}}(s) \Longrightarrow$ Good data crucial



- We have to use a large number (>80) of data sets $\Longrightarrow$ Statistically correct treatment/combination of data sets important

How to Combine data sets — “Clustering”

1. We model the true value of R by a piecewise-constant $\overline{R}_m$ within a Cluster of a given (min.) size.



2. Construct the χ^2 function as

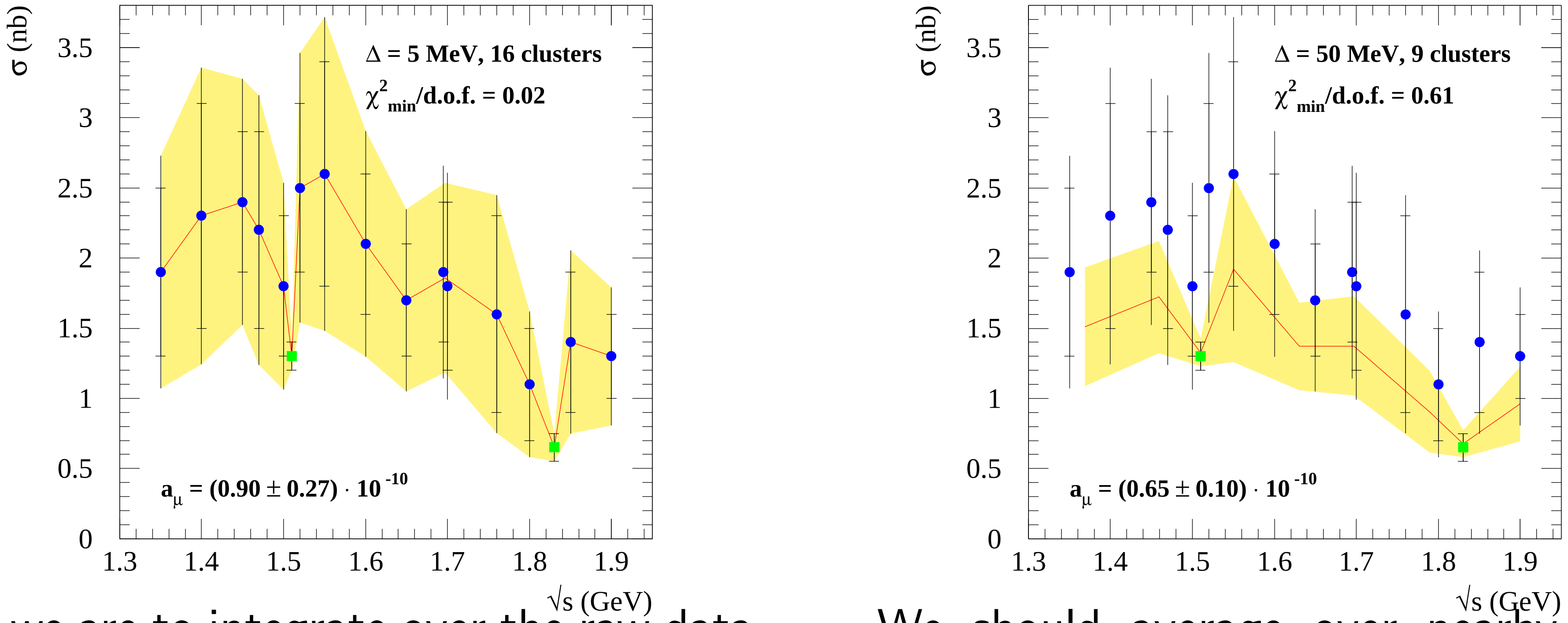
$$\chi^2(\overline{R}_m, f_k) = \sum_{k=1}^{\#\text{ofexp.}} \left(\frac{1-f_k}{df_k}\right)^2 + \sum_{m=1}^{\#\text{ofClus.}} \sum_{i=1}^{N_{\{k,m\}}} \left(\frac{R_i^{\{k,m\}} - f_k \overline{R}_m}{dR_i^{\{k,m\}}}\right)^2$$

   from the raw data $R_i^{\{k,m\}} \pm dR_i^{\{k,m\}}$ and the **normalization uncertainty** of the k-th exp df_k.

3. Minimize it w. r. t. $\overline{R}_m$ and f_k.

Combining data sets (“Clustering”) — Toy Example

Suppose we have two data sets — one good (green), the other poor (blue)



If we are to integrate over the raw data, the result would be like this — we are:

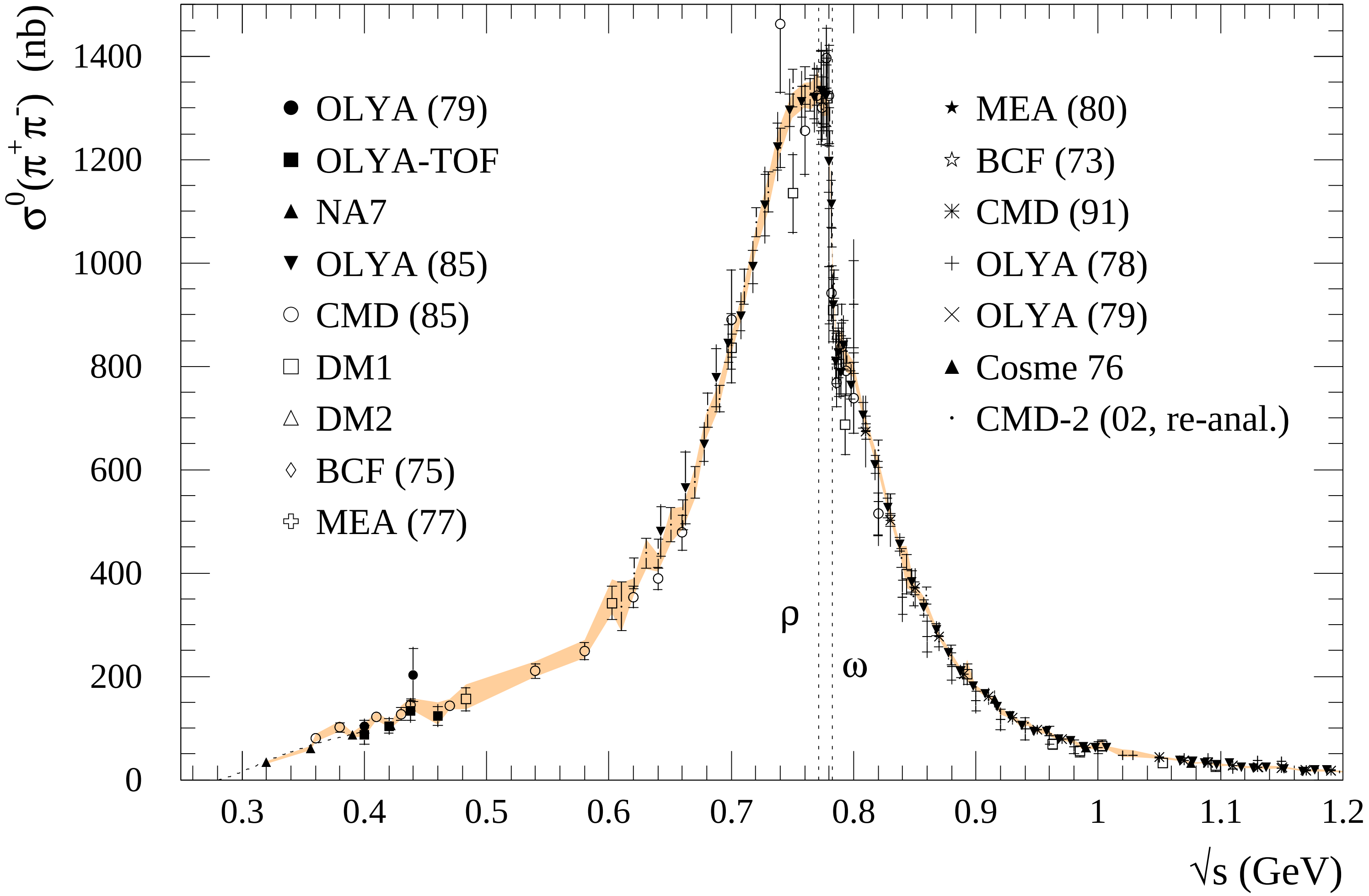
- overestimating the error
- overestimating the mean (in this case)

We should average over nearby data points (“clustering”)

Advantages

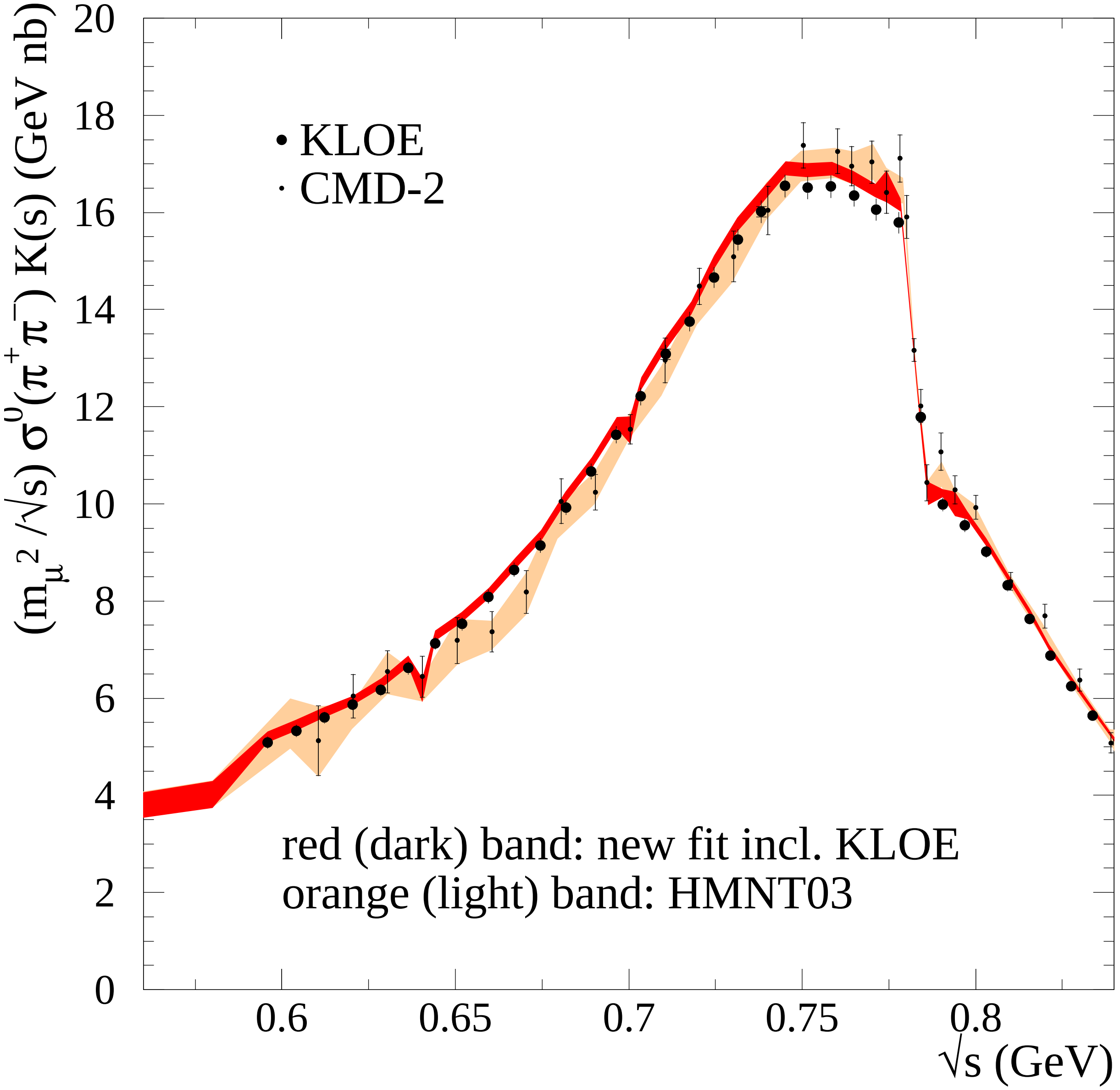
- overall normalization uncertainty of the poor data set fixed by the good one
- combining effect (N times data in one bin $\Longrightarrow$ error reduced by a factor of $1/\sqrt{N}$)

Clustering — Real Data ($e^+e^- \to \pi^+\pi^-$)



$\pi^+\pi^-$: by far the **most important** channel — **73 %** of total $a_\mu^{\mathrm{had,LO}}$

Comments on the KLOE data



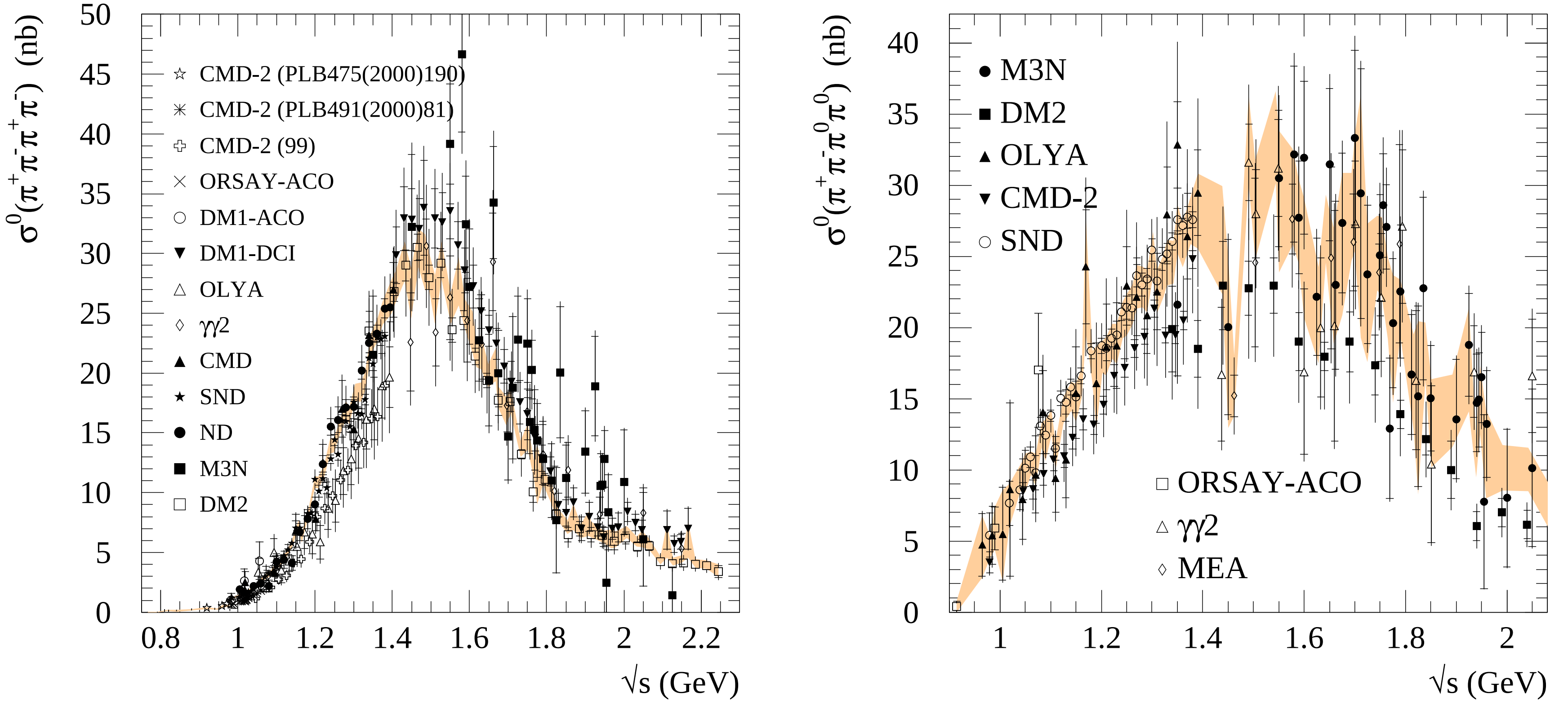
New data of the pion form factor appeared from **KLOE** (`hep-ex/0407048`) using $e^+e^- \to \pi^+\pi^-\gamma$

✓ Good quality data (small error)

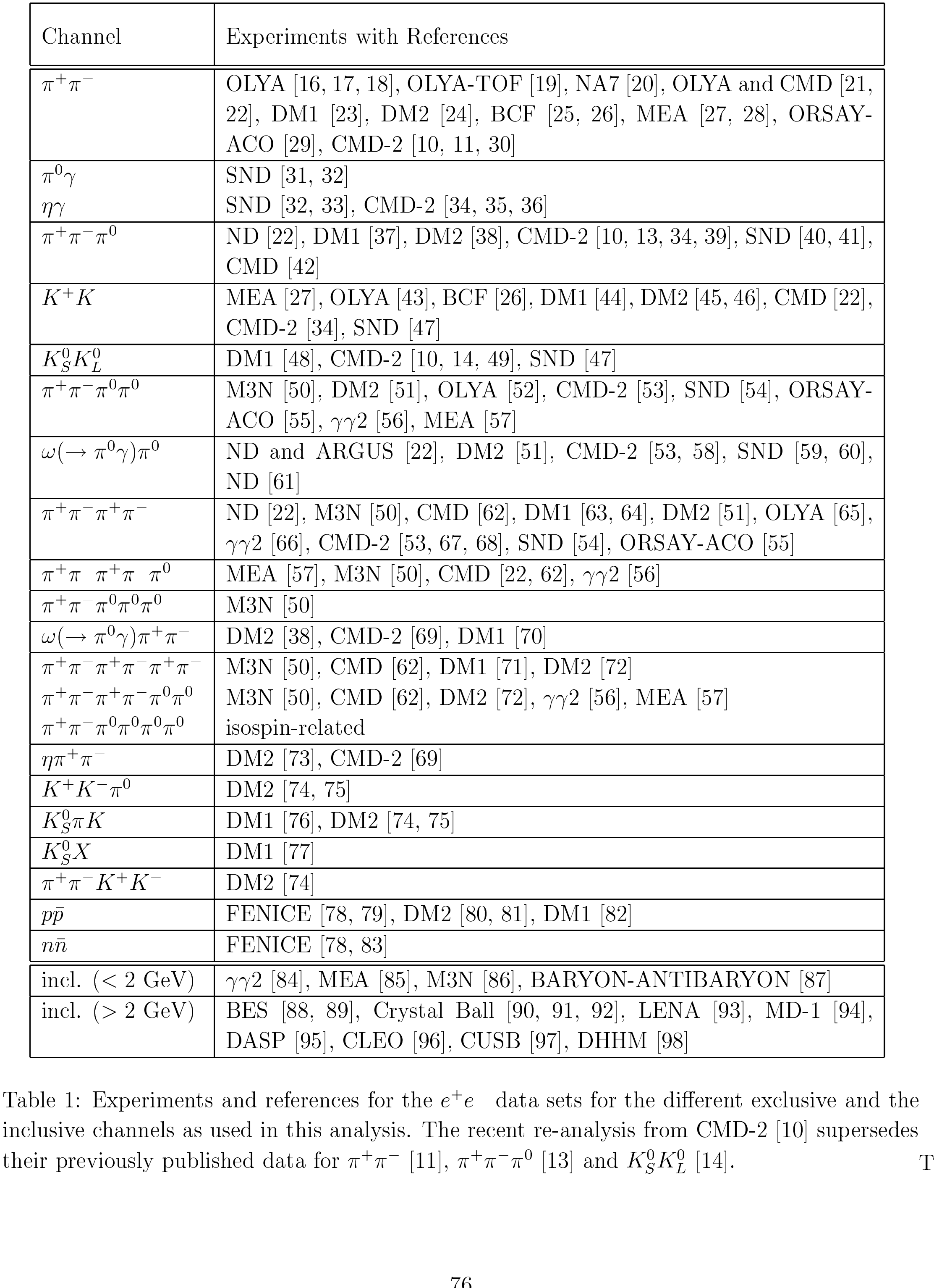
× **Inconsistent shape** with CMD-2 ⟵ **not yet understood why**

We (HMNT) combined them only after integrating over the e^+e^- data and the KLOE data separately.

Clustering — More “Difficult” Channels (e.g. $e^+e^- \to 4\pi$)

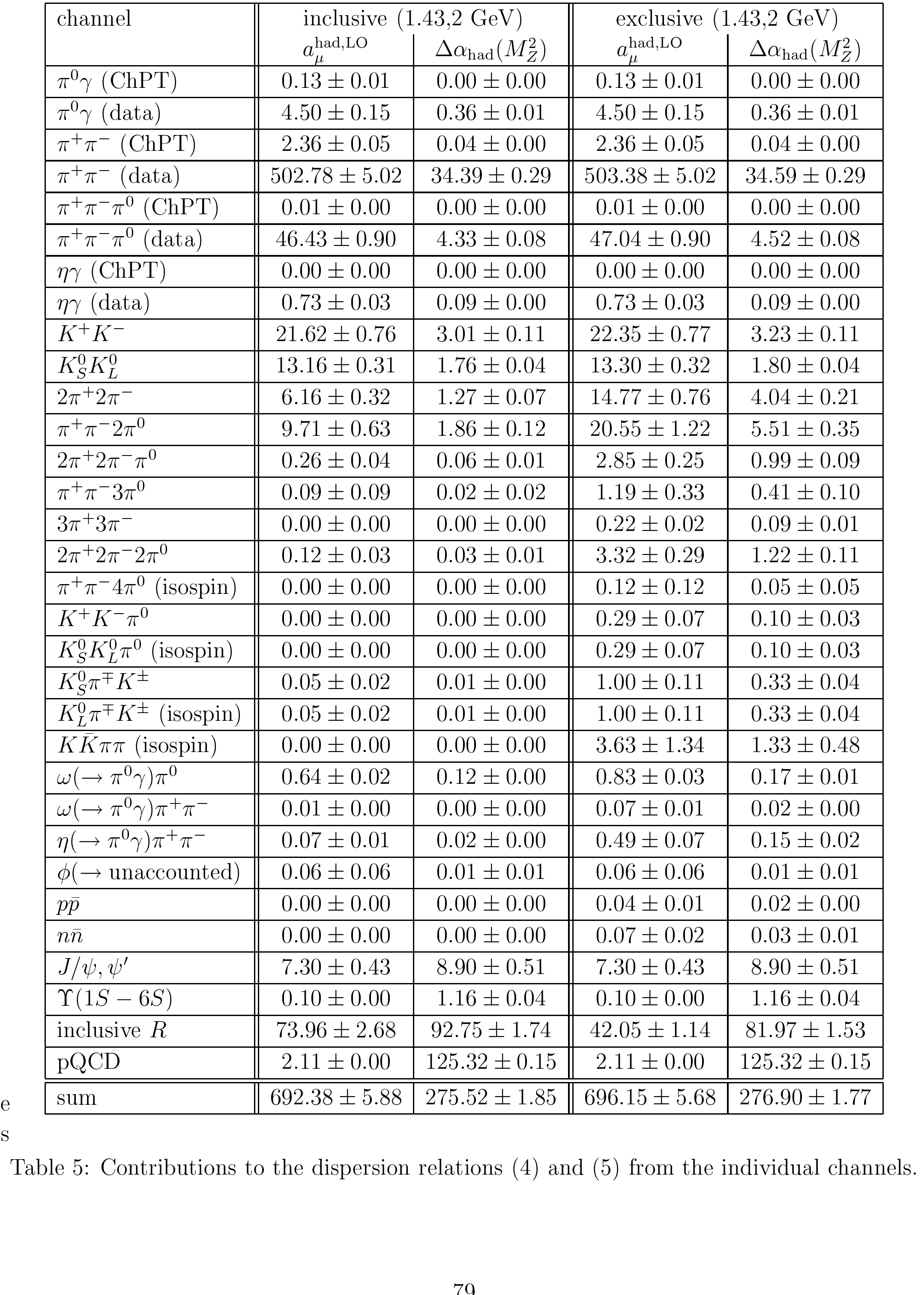


$2\pi^+2\pi^-$ and **$\pi^+\pi^-2\pi^0$**: $\chi^2_{\rm min}/{\rm d.o.f}$ not good (2.00 and 1.28) — we have inflated the error by a factor of $\sqrt{\chi^2_{\rm min}/{\rm d.o.f}}$



Channel	Experiments with References
$\pi^+\pi^-$	OLYA [16, 17, 18], OLYA-TOF [19], NA7 [20], OLYA and CMD [21, 22], DM1 [23], DM2 [24], BCF [25, 26], MEA [27, 28], ORSAY-ACO [29], CMD-2 [10, 11, 30]
$\pi^0\gamma$	SND [31, 32]
$\eta\gamma$	SND [32, 33], CMD-2 [34, 35, 36]
$\pi^+\pi^-\pi^0$	ND [22], DM1 [37], DM2 [38], CMD-2 [10, 13, 34, 39], SND [40, 41], CMD [42]
K^+K^-	MEA [27], OLYA [43], BCF [26], DM1 [44], DM2 [45, 46], CMD [22], CMD-2 [34], SND [47]
$K^0_SK^0_L$	DM1 [48], CMD-2 [10, 14, 49], SND [47]
$\pi^+\pi^-\pi^0\pi^0$	M3N [50], DM2 [51], OLYA [52], CMD-2 [53], SND [54], ORSAY-ACO [55], $\gamma\gamma$2 [56], MEA [57]
$\omega(\to \pi^0\gamma)\pi^0$	ND and ARGUS [22], DM2 [51], CMD-2 [53, 58], SND [59, 60], ND [61]
$\pi^+\pi^-\pi^+\pi^-$	ND [22], M3N [50], CMD [62], DM1 [63, 64], DM2 [51], OLYA [65], $\gamma\gamma$2 [66], CMD-2 [53, 67, 68], SND [54], ORSAY-ACO [55]
$\pi^+\pi^-\pi^+\pi^-\pi^0$	MEA [57], M3N [50], CMD [22, 62], $\gamma\gamma$2 [56]
$\pi^+\pi^-\pi^0\pi^0\pi^0$	M3N [50]
$\omega(\to \pi^0\gamma)\pi^+\pi^-$	DM2 [38], CMD-2 [69], DM1 [70]
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	M3N [50], CMD [62], DM1 [71], DM2 [72]
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	M3N [50], CMD [62], DM2 [72], $\gamma\gamma$2 [56], MEA [57]
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$	isospin-related
$\eta\pi^+\pi^-$	DM2 [73], CMD-2 [69]
$K^+K^-\pi^0$	DM2 [74, 75]
$K^0_S\pi K$	DM1 [76], DM2 [74, 75]
K^0_SX	DM1 [77]
$\pi^+\pi^-K^+K^-$	DM2 [74]
$p\bar{p}$	FENICE [78, 79], DM2 [80, 81], DM1 [82]
$n\bar{n}$	FENICE [78, 83]
incl. (< 2 GeV)	$\gamma\gamma$2 [84], MEA [85], M3N [86], BARYON-ANTIBARYON [87]
incl. (> 2 GeV)	BES [88, 89], Crystal Ball [90, 91, 92], LENA [93], MD-1 [94], DASP [95], CLEO [96], CUSB [97], DHHM [98]

Table 1: Experiments and references for the e^+e^- data sets for the different exclusive and the inclusive channels as used in this analysis. The recent re-analysis from CMD-2 [10] supersedes their previously published data for $\pi^+\pi^-$ [11], $\pi^+\pi^-\pi^0$ [13] and $K^0_SK^0_L$ [14].



channel	inclusive (1.43,2 GeV)		exclusive (1.43,2 GeV)	
	$a_\mu^{\rm had,LO}$	$\Delta\alpha_{\rm had}(M_Z^2)$	$a_\mu^{\rm had,LO}$	$\Delta\alpha_{\rm had}(M_Z^2)$
$\pi^0\gamma$ (ChPT)	0.13 ± 0.01	0.00 ± 0.00	0.13 ± 0.01	0.00 ± 0.00
$\pi^0\gamma$ (data)	4.50 ± 0.15	0.36 ± 0.01	4.50 ± 0.15	0.36 ± 0.01
$\pi^+\pi^-$ (ChPT)	2.36 ± 0.05	0.04 ± 0.00	2.36 ± 0.05	0.04 ± 0.00
$\pi^+\pi^-$ (data)	502.78 ± 5.02	34.39 ± 0.29	503.38 ± 5.02	34.59 ± 0.29
$\pi^+\pi^-\pi^0$ (ChPT)	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00	0.00 ± 0.00
$\pi^+\pi^-\pi^0$ (data)	46.43 ± 0.90	4.33 ± 0.08	47.04 ± 0.90	4.52 ± 0.08
$\eta\gamma$ (ChPT)	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\gamma$ (data)	0.73 ± 0.03	0.09 ± 0.00	0.73 ± 0.03	0.09 ± 0.00
K^+K^-	21.62 ± 0.76	3.01 ± 0.11	22.35 ± 0.77	3.23 ± 0.11
$K^0_SK^0_L$	13.16 ± 0.31	1.76 ± 0.04	13.30 ± 0.32	1.80 ± 0.04
$2\pi^+2\pi^-$	6.16 ± 0.32	1.27 ± 0.07	14.77 ± 0.76	4.04 ± 0.21
$\pi^+\pi^-2\pi^0$	9.71 ± 0.63	1.86 ± 0.12	20.55 ± 1.22	5.51 ± 0.35
$2\pi^+2\pi^-\pi^0$	0.26 ± 0.04	0.06 ± 0.01	2.85 ± 0.25	0.99 ± 0.09
$\pi^+\pi^-3\pi^0$	0.09 ± 0.09	0.02 ± 0.02	1.19 ± 0.33	0.41 ± 0.10
$3\pi^+3\pi^-$	0.00 ± 0.00	0.00 ± 0.00	0.22 ± 0.02	0.09 ± 0.01
$2\pi^+2\pi^-2\pi^0$	0.12 ± 0.03	0.03 ± 0.01	3.32 ± 0.29	1.22 ± 0.11
$\pi^+\pi^-4\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.12 ± 0.12	0.05 ± 0.05
$K^+K^-\pi^0$	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K^0_SK^0_L\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K^0_S\pi^\mp K^\pm$	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K^0_L\pi^\mp K^\pm$ (isospin)	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K\bar{K}\pi\pi$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	3.63 ± 1.34	1.33 ± 0.48
$\omega(\to \pi^0\gamma)\pi^0$	0.64 ± 0.02	0.12 ± 0.00	0.83 ± 0.03	0.17 ± 0.01
$\omega(\to \pi^0\gamma)\pi^+\pi^-$	0.01 ± 0.00	0.00 ± 0.00	0.07 ± 0.01	0.02 ± 0.00
$\eta(\to \pi^0\gamma)\pi^+\pi^-$	0.07 ± 0.01	0.02 ± 0.00	0.49 ± 0.07	0.15 ± 0.02
$\phi(\to$ unaccounted)	0.06 ± 0.06	0.01 ± 0.01	0.06 ± 0.06	0.01 ± 0.01
$p\bar{p}$	0.00 ± 0.00	0.00 ± 0.00	0.04 ± 0.01	0.02 ± 0.00
$n\bar{n}$	0.00 ± 0.00	0.00 ± 0.00	0.07 ± 0.02	0.03 ± 0.01
$J/\psi, \psi'$	7.30 ± 0.43	8.90 ± 0.51	7.30 ± 0.43	8.90 ± 0.51
$\Upsilon(1S - 6S)$	0.10 ± 0.00	1.16 ± 0.04	0.10 ± 0.00	1.16 ± 0.04
inclusive R	73.96 ± 2.68	92.75 ± 1.74	42.05 ± 1.14	81.97 ± 1.53
pQCD	2.11 ± 0.00	125.32 ± 0.15	2.11 ± 0.00	125.32 ± 0.15
sum	692.38 ± 5.88	275.52 ± 1.85	696.15 ± 5.68	276.90 ± 1.77

Table 5: Contributions to the dispersion relations (4) and (5) from the individual channels.

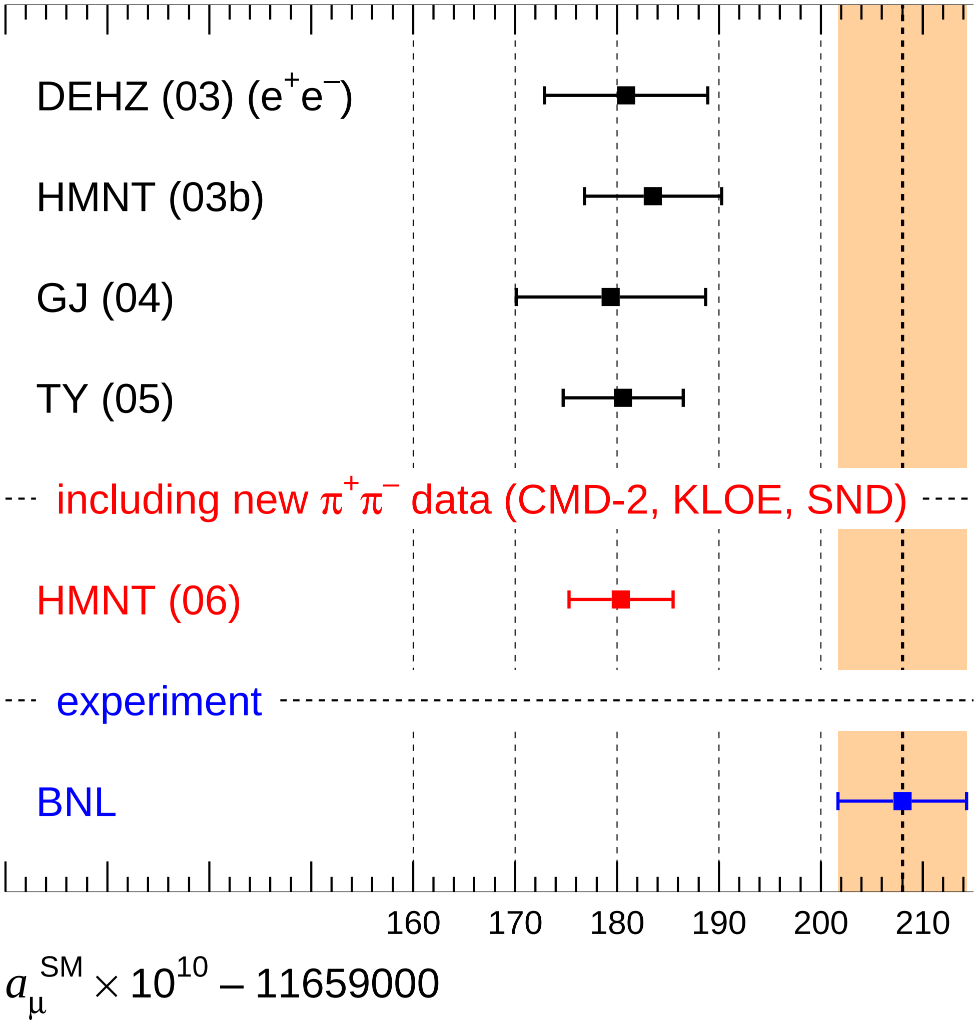
Our Evaluation of $a_\mu^{\text{had,LO}}$ and Breakdown

energy range (GeV)	$a_\mu^{\text{had,LO}} \times 10^{10}$	comments
$m_\pi \ldots 0.32$	2.49 ± 0.05	chiral PT
$0.32 \ldots 1.43$	602.03 ± 3.19	sum of exclusive data
$1.43 \ldots 2.00$	32.05 ± 2.43	inclusive measurements
$2.00 \ldots 11.09$	42.75 ± 1.08	inclusive measurements
J/ψ and $\psi(2S)$	7.90 ± 0.16	narrow width approx.
$\Upsilon(1S-6S)$	0.10 ± 0.00	narrow width approx.
$11.09 \ldots \infty$	2.11 ± 0.00	pQCD
$\sum$ of all	$689.44 \pm 4.17_{\text{exp}}$	

★ The sum is dominated by the contribution from low energies, $\sqrt{s} \lesssim 1.4\text{GeV}$. (Roughly 600 out of 700)

★ $a_\mu^{\text{had, NLO}}$ can be evaluated similarly. Our result: $a_\mu^{\text{had, NLO}} = (-9.79 \pm 0.09) \times 10^{-10}$.

$a_\mu^{\text{had,LO}}$ combined with the other contributions to a_μ^{SM}



HMNT, hep-ph/0611102

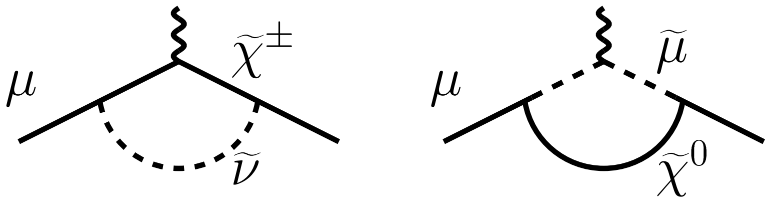
- Our results: consistent with previous results with smaller error

✓ $\delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.6 \pm 8.1) \times 10^{-10}$: **3.4$\sigma$ discrepancy**

SUSY Contributions?

Is the 3.4σ deviation due to SUSY?

Dominant **SUSY contributions**:



which is, very roughly, given by

$$a_\mu^{\rm SUSY} = ({\rm sgn}\ \mu)\frac{\alpha(M_Z)}{8\pi \sin^2\theta_W}\frac{m_\mu^2}{\widetilde{m}^2}\tan\beta,$$

where $\widetilde{m}$ is the SUSY scale.

Numerically,

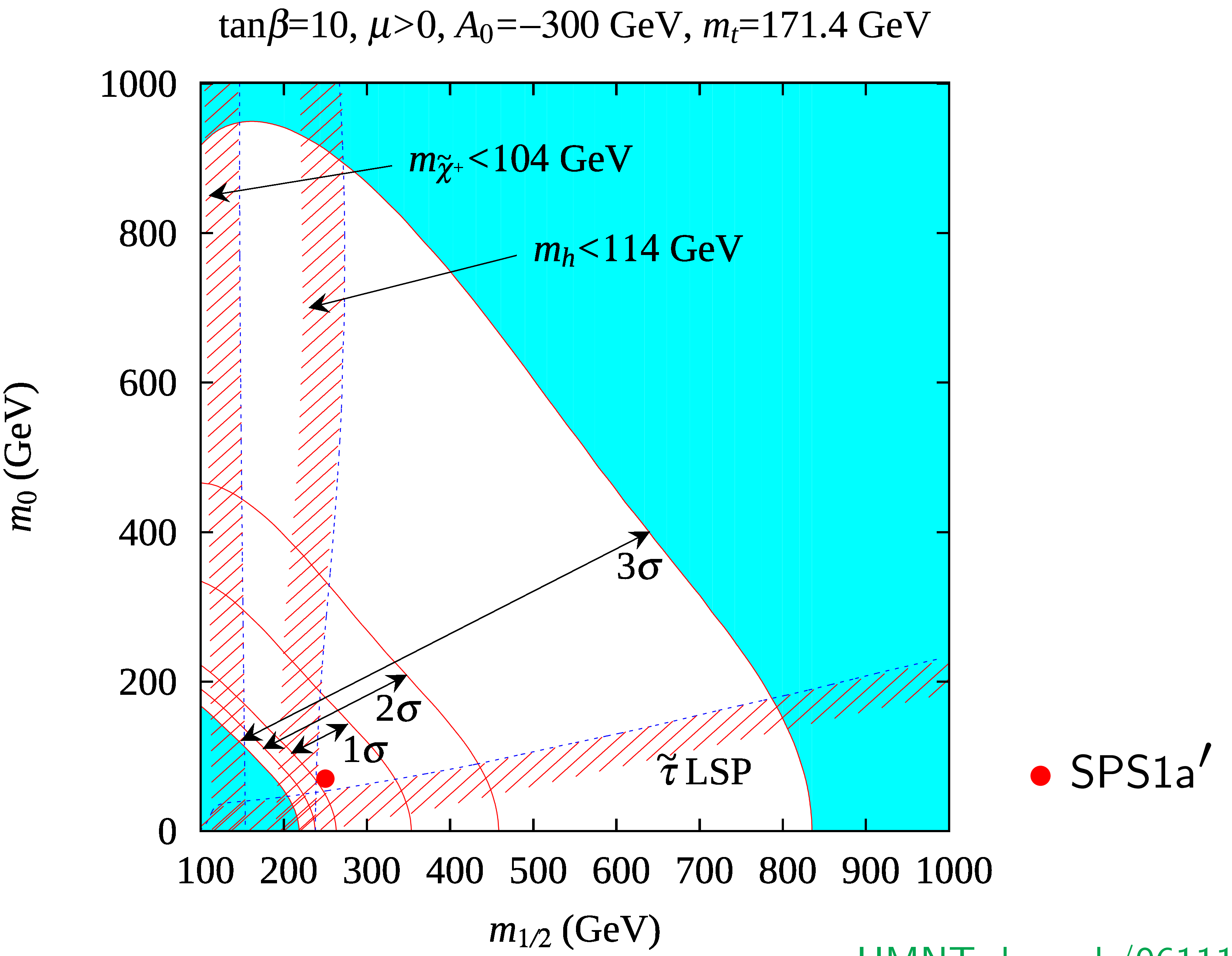
$$a_\mu^{\rm SUSY} = ({\rm sgn}\mu) \times 13 \times 10^{-10} \times \left(\frac{100{\rm GeV}}{\widetilde{m}}\right)^2 \tan\beta$$

In order for this to be $11.4 \le a_\mu^{\rm SUSY} \times 10^{10} \le 43.8$ (2 σ range),

$$\boxed{\widetilde{m} = \mathbf{170 - 760}\ \text{GeV}}$$

for $\tan\beta = 10 - 50$. (**Rough estimates**)

Impact on mSUGRA Parameter Space (Example)



HMNT, hep-ph/0611102

Summary

✓ The largest uncertainty in a_μ: still from the **LO hadronic** contribution.

★ **Our results**: **3.4 σ** deviation from experiment. $\Longrightarrow$ **SUSY contribution?**

★ According to the paper by Benayoun et al, the consistency problem between the e^+e^- and τ data is no longer observed, which supports our results based on the e^+e^- data.

▶ Waiting for new precise data from the radiative return at BaBar and Belle in multi-pion channels.

▶ New data on the pion form factor appeared from **KLOE**, but there is some inconsistency in shape with CMD-2 and SND data, which is yet to be understood.

▶ proposal at **BNL** (E969): If approved, a factor of **2.5** (or more) improvement expected.

▶ planned measurement of a_μ at **J-PARC**: a factor of **4 – 6** improvement expected.