SOME PROBLEMS of STRANGE BARYONS SPECTROSCOPY: CHIRAL SOLITONS VERSUS QUARK MODELS

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1. In spite of (or due to?) recent dramatic events the studies of baryons spectrum - strange, nonstrange, ...- remain to be very actual for accelerator physics. Discovery of baryon states besides well established (octet, decuplet...) could help in the progress in understanding hadrons structure.

In the absence of the complete theory of strong interactions there are different approaches and models; each has some advantages and certain drawbacks.

Interpretation of hadrons spectra in terms of quark models (QM) is widely accepted, QM are "most successfull tool for the classification and interpretation" (R.Jaffe) of hadrons spectrum. QM are to large extent phenomenological since there is no regular methods of solving relativistic many-body problem. The number of constituents (e.g. additional $q\bar{q}$-pairs) is not fixed in a true relativistic theory.

Alternative approaches, in particular, chiral soliton approach (CSA) has certain advantages. It is based on few principles and ingrediemts incorporated in the model lagrangian.
Baryons and baryonic systems are considered on equal footing (the look "from outside"). CSA looks like a theory, but still it is a model. Some elements of phenomenology are present necessarily in CSA as well. Results obtained within CSA mimic some features of baryons spectrum within quark models due to Gell-Mann - Okubo relations.

2. The CSA is based on few principles and ingredients incorporated in the *truncated* **effective chiral lagrangian**:

$$L^{eff} = -\frac{F_\pi^2}{16} Tr l_\mu l_\mu + \frac{1}{32e^2} Tr[l_\mu l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{8} Tr(U + U^\dagger - 2) + ...,$$

$l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $\in SU(3)$- unitary matrix depending on chiral fields, m_π is pion mass, F_π-pion decay constant, e - the only parameter of the model.

The mass term $\sim F_\pi^2 m_\pi^2$, changes asymptotics of the profile f and the structure of multiskyrmions at large B. For the $SU(2)$ case

$$U = cos f + i\,(\vec{n}\vec{\tau})\,sin\,f,$$

unit $\vec{n}$ depends on 2 functions, α, β. Three profiles $\{f,\ \alpha,\ \beta\}(x,y,z)$ parametrize the unit vector on the 3-sphere S^3.

The soliton is configuration of chiral fields, possessing topological charge identified with the baryon number B (Skyrme, 1961):

$$B = \frac{1}{2\pi^2}\int s_f^2 s_\alpha I\,[(f,\alpha,\beta)/(x,y,z)]\,d^3r$$

where I is the Jacobian of the coordinates transformation, $s_f = sin\,f$. So, the quantity B shows how many times S^3 is covered when integration over R^3 is made.

Recall that surface of the unit sphere S^3 equals

$$\int s_f^2 s_\alpha df d\alpha d\beta = 2\pi^2.$$

Masses, binding energies of classical configurations, moments of inertia Θ_I, Θ_J and some other characteristics of chiral solitons contain implicitly information about interaction between baryons. Minimization of the mass functional M_{class} provides 3 profiles and allows to calculate moments of inertia, etc.

3. The observed spectrum of states is obtained by means of quantization procedure and depends on quantum numbers and moments of inertia, Σ-term (Γ), etc.
In $SU(2)$ case, the rigid rotator model (RRM) is most effective and successfull in describing the properties of nucleons, Δ, and also "symmetry energy" of lighter nuclei.

In the $SU(3)$ case the mass formula takes place, also for RRM

$$M(p,q,Y,I,J) = M_{cl} + \frac{K(p,q)}{2\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \delta M,$$

$$\sim N_c \qquad \sim 1 \qquad \sim N_c^{-1} \qquad \sim 1,$$

it is in fact expansion in powers of $1/N_c$. Some paradox is in the fact that total splitting of the whole multiplet is $\sim N_c$.

Mass splittings δM are due to term in the lagrangian

$$\mathcal{L}_M \simeq \tilde{m}_K^2 \Gamma \frac{s_\nu^2}{2},$$

ν is the angle of rotation into strange direction, $\tilde{m}_K^2 = F_K^2 m_K^2 / F_\pi^2 - m_\pi^2$ includes $SU(3)$-symmetry violation in flavor decay constants, $\Gamma \sim 5\, Gev^{-1} \sim \Sigma$, moments of inertia $\Theta_\pi \sim (5-6) Gev^{-1}$, $\Theta_K \sim (2-3) Gev^{-1}$. $\Theta \sim N_c$.

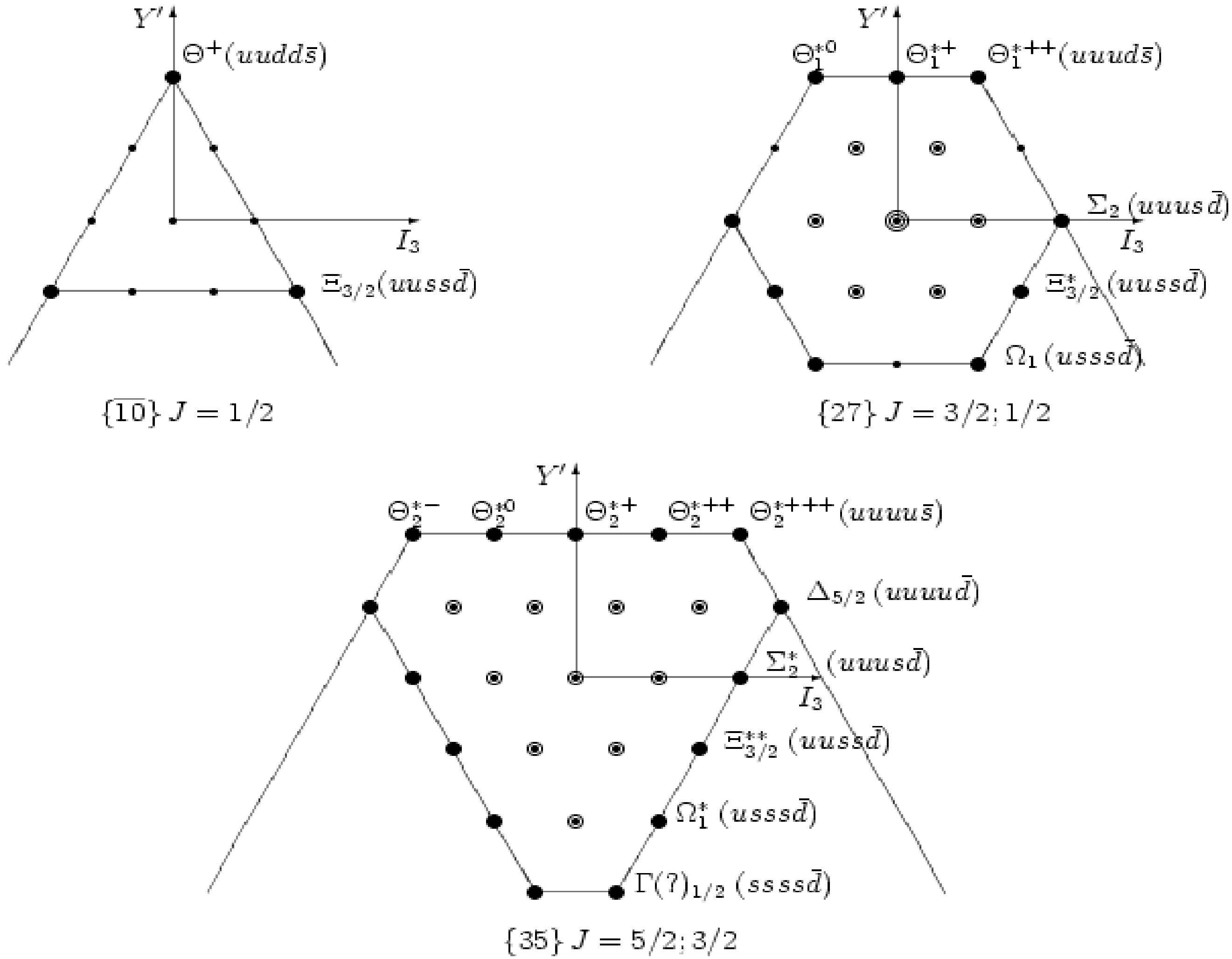


Figure 1: The $I_3 - Y'$ diagrams ($Y' = S + 1$) for multiplets of pentaquark baryons, antidecuplet, $\{27\}$- and $\{35\}$-plets. For $N > 3$ these diagrams should be extended within long lines, as shown in the picture. Quark contents are given for manifestly exotic states, when $N = 3$.

"Strangeness contents"

$$C_S = < s_\nu^2/2 >_B$$

can be calculated exactly with the help of wave functions in $SU(3)$ configuration space, for arbitrary number of colors N_c.

Some examples of values of S_C at arbitrary number of colors N_c:

$$C_S(\text{"}N\text{"}) = \frac{2(N_c+4)}{(N_c+3)(N_c+7)}, \; C_S(\text{"}\Xi\text{"}) = \frac{4}{N_c+7},$$

$$C_S(\text{"}\Delta\text{"}) = \frac{2(N_c+4)}{(N_c+1)(N_c+9)},$$

$$C_S(\text{"}\Theta\text{"}) = \frac{3}{N_c+9}, \; C_S(\text{"}\Phi\text{"}) = \frac{6N_c+9}{(N_c+3)(N_c+9)},$$

$$C_S(\text{"}\Theta_1\text{"}) = \frac{3N_c+23}{(N_c+5)(N_c+11)}.$$

Approximately at large N_c

$$C_S \simeq \frac{2+|S|}{N_c}.$$

The Gell-Mann - Okubo formula takes place in the form

$$C_S = -A(p,q)\,Y - B(p,q)\left[Y^2/4 - \vec{I}^2\right] + C(p,q),$$

$A_(p,q), B_(p,q), C_(p,q)$ depend on particular $SU(3)$ multiplet. For the "octet"

$$A(\text{"}8\text{"}) = \frac{N_c+2}{(N_c+3)(N_c+7)}, \quad B(\text{"}8\text{"}) = \frac{2}{(N_c+3)(N_c+7)},$$

$$C(\text{"}8\text{"}) = \frac{3}{(N_c+7)}.$$

If we try to make expansion in $1/N_c$, then parameter is $\sim 7/N_c$. For "decuplet" and "antidecuplet" expansion parameter is $\sim 9/N_c$ and becomes worse for greater multiplets, "27"-plet, "35"-plet, etc. Apparently, for realistic world with $N_C = 3$ the $1/N_c$ expansion *does not work*.

Any chain of states connected by relation $I = C' \pm Y/2$ reveals linear dependence on hypercharge (strangeness), so, CSA *mimics the quark model* with effective strange quark mass

$$m_S^{eff} \sim \tilde{m}_K^2 \Gamma [A(p,q) \mp 3B(p,q)/2],$$

for decuplet (antidecuplet). This is valid if the FSB is included in the lowest order of perturbation theory.

At large N_c

$$m_S^{eff} \sim \tilde{m}_K^2 \Gamma / N_c,$$

too much, $\sim 0.6\, GeV$ if extrapolated to $N_c = 3$.

If we make expansion in RRM, we obtain for the "octet" of baryons

$$\delta M_N = 2\tilde{m}_K^2 \frac{\Gamma}{N_c}\Big(1 - \frac{6}{N_c}\Big)$$

$$\dots\dots\dots$$

$$\delta M_\Xi = \tilde{m}_K^2 \frac{\Gamma}{N_c}\Big(4 - \frac{28}{N_c}\Big),$$

Within the **bound state model** (BSM) (Callan, Klebanov, Riska, Scoccola,.. '85, '86) anti-kaon is bound by $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{HFS}$$

where flavor and antiflavor excitation energies

$$\omega_S = N_c(\mu - 1)/8\Theta_K, \;\; \omega_{\bar{F}} = N_c(\mu + 1)/8\Theta_K,$$

$$\mu = \sqrt{1 + \bar{m}_K^2/M_0^2} \simeq 1 + \frac{\bar{m}_K^2}{2M_0^2},$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_K) \sim N_c^0, \quad \mu \sim N_c^0.$$

The hyperfine splitting correction depending on hyperfine splitting constants c and $\bar{c}$, and "strange isospin" $I_S = |S|/2$ equals

$$\Delta M_{HFS} = \frac{J(J+1)}{2\Theta_\pi} + \frac{(c_S - 1)[J(J+1) - I(I+1)] + (\bar{c}_S - c_S)I_S(I_S+1)}{2\Theta_\pi}$$

and is small at large N_c, $\sim 1/N_c$, and for heavy flavors. For anti-flavor (positive strangeness) certain changes should be done: $\omega_S \longrightarrow \omega_{\bar{S}}$ and $c_S \longrightarrow c_{\bar{S}}$ in the last term.

In this way we obtain for the "octet"

$$\delta M_N = 2\tilde{m}_K^2 \frac{\Gamma}{N_c}$$

$$\delta M_\Xi = \tilde{m}_K^2 \frac{\Gamma}{N_c}\Big(4 - \frac{4}{N_c}\Big),$$

Total splitting of the "octet"

$$\Delta_{tot}("8", BSM) = \tilde{m}_K^2 \frac{\Gamma}{N_c}\Big(3 - \frac{4}{N_c}\Big),$$

$$\Delta_{tot}("8", RRM) = \tilde{m}_K^2 \frac{\Gamma}{N_c}\Big(3 - \frac{22}{N_c}\Big).$$

In BSM mass splittings are bigger than in RRM.

The RRM used for prediction of pentaquarks (M.Praszalowicz, H.Walliser, D.Diakonov,..,, H.Weigel) *different* from the BSM model, used in N.Itzhaki et al (2004) to disavow the Θ^+.

The case of exotic $S = +1$ Θ hyperons is especially interesting. In BSM we obtain

$$M_{\Theta_0,J=1/2} = M_{cl} + \frac{2N_c+3}{4\Theta_K} + \frac{3}{8\Theta_\pi} + \bar{m}_K^2\Gamma\Big(\frac{3}{N_c} - \frac{9}{N_c^2}\Big)$$

$$M_{\Theta_1,J=3/2} = M_{cl} + \frac{2N_c+1}{4\Theta_K} + \frac{15}{8\Theta_\pi} + \bar{m}_K^2\Gamma\Big(\frac{3}{N_c} - \frac{7}{N_c^2}\Big)$$

$$M_{\Theta_2,J=5/2} = M_{cl} + \frac{2N_c-1}{4\Theta_K} + \frac{35}{8\Theta_\pi} + \bar{m}_K^2\Gamma\Big(\frac{3}{N_c} - \frac{5}{N_c^2}\Big)$$

The terms $\sim 1/\Theta_K$ agree with the difference of $C_2(SU_3)$ for anti-decuplet, $\{27\}$- and $\{35\}$-plets, as obtained in RRM.

We should compare this with the mass splitting correction from RRM:

$$\delta M_{\Theta_0,J=1/2} = \bar{m}_K^2\Gamma\Big(\frac{3}{N_c} - \frac{27}{N_c^2}\Big)$$

$$\delta M_{\Theta_1,J=3/2} = \bar{m}_K^2\Gamma\Big(\frac{3}{N_c} - \frac{25}{N_c^2}\Big)$$

$$\delta M_{\Theta_2,J=5/2} = \bar{m}_K^2\Gamma\Big(\frac{3}{N_c} - \frac{23}{N_c^2}\Big)$$

and again considerable difference takes place.

The addition of the term to the BSM result,*possible due to normal ordering ambiguity present in BSM* (I.Klebanov, VBK, 2005)

$$\Delta M_{BSM} = -6\bar{m}_K^2 \frac{\Gamma}{N_c^2}(2 + |S|)$$

brings results of RRM and BSM in agreement - for nonexotic and exotic states. This procedure looks not quite satisfactorily: if we believe to RRM, why we need BSM at all? Anyway, RRM and BSM in its accepted form are *different models*.

The rotation-vibration approach (RVA) developed by H.Weigel and H.Walliser in 2005 unifies RRM and BSM in some way, and Θ^+ has been confirmed with somewhat higher energy and considerable width ($\Gamma_\Theta \sim 50\, MeV$).

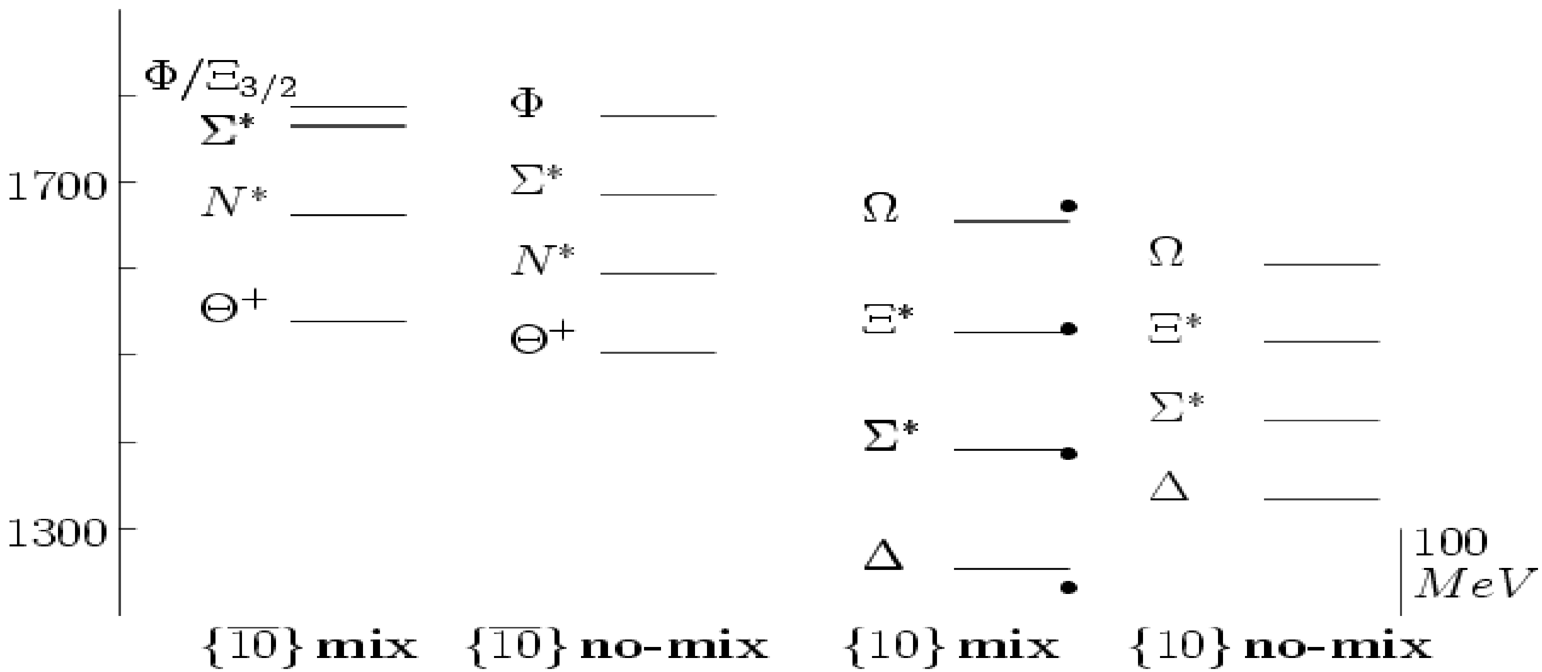


Fig. *Influence of the configuration mixing* (H.Yabu, K.Ando, 1988) on the mass splitting within antidecuplet and decuplet of baryons, RR model (variant by H.Walliser, VBK, 2003):

Strange, or kaonic inertia Θ_K contains important contribution due to flavor symmetry breaking in meson decay constants, $F_K/F_\pi \simeq 1.23$:

$$\Theta_K = \frac{1}{8}\int(1-c_f)\left[F_K^2 + (Skyrme\ term)\right]d^3r.$$

For anti-decuplet mixing decreases slightly the total splitting, and pushes N^* and Σ^* toward higher energy. Mixing with components of the octet is important. Apparent contradiction with simplest assumption of equality of masses of strange quarks and antiquarks $m(s) = m(\bar{s})$.

For decuplet mixing increases total splitting considerably, but approximate **equidistancy still remains**! Mixing with components of $\{27\}$-plet is important. Data - black points.

A note for the QM: states with different numbers of $q\bar{q}$ pairs can mix, and such mixing *should be taken into account*.

5. It is possible to make comparison of CSA results with expectations from simple quark model in *pentaquark* approximation (projection of CSM on QM).

The masses m_s, $m_{\bar{s}}$ and $m(s\bar{s})$ come into play.

$\vert\overline{10}, 2, 0 >$	$\vert\overline{10}, 1, \frac{1}{2} >$	$\vert\overline{10}, 0, 1 >$	$\vert\overline{10}, -1, \frac{3}{2} >$		
$m_{\bar{s}}$	$2m_{s\bar{s}}/3$	$m_s + m_{s\bar{s}}/3$	$2m_s$		
564	655	745	836		
600	722	825	847		
$\vert 27, 2, 1 >$	$\vert 27, 1, \frac{3}{2} >$	$\vert 27, 0, 2 >$	$\vert 27, -1, \frac{3}{2} >$	$\vert 27, -2, 1 >$	
$m_{\bar{s}}$	$m_{s\bar{s}}/2$	m_s	$2m_s$	$3m_s$	
733	753	772	889	1005	
749	887	779	911	1048	
$\vert 35, 2, 2 >$	$\vert 35, 1, \frac{5}{2} >$	$\vert 35, 0, 2 >$	$\vert 35, -1, \frac{3}{2} >$	$\vert 35, -2, 1 >$	$\vert 35, -3, \frac{1}{2} >$
$m_{\bar{s}}$	0	m_s	$2m_s$	$3m_s$	$4m_s$
1152	857	971	1084	1197	1311
1122	853	979	1107	1236	1367

Strange quark (antiquark) masses contributions and calculation results within RRM with and without configuration mixing.

Simple relations can be obtained from this Table for effective $s-$quark masses:

from the total splitting of antidecuplet

$$[2m_s - m_{\bar{s}}]_{\overline{10}} = 247\, MeV\ (272\, Mev).$$

from splittings within 27-plet

$$[m_s - m_{\bar{s}}]_{27} = 30\, Mev\ (39\, Mev),$$

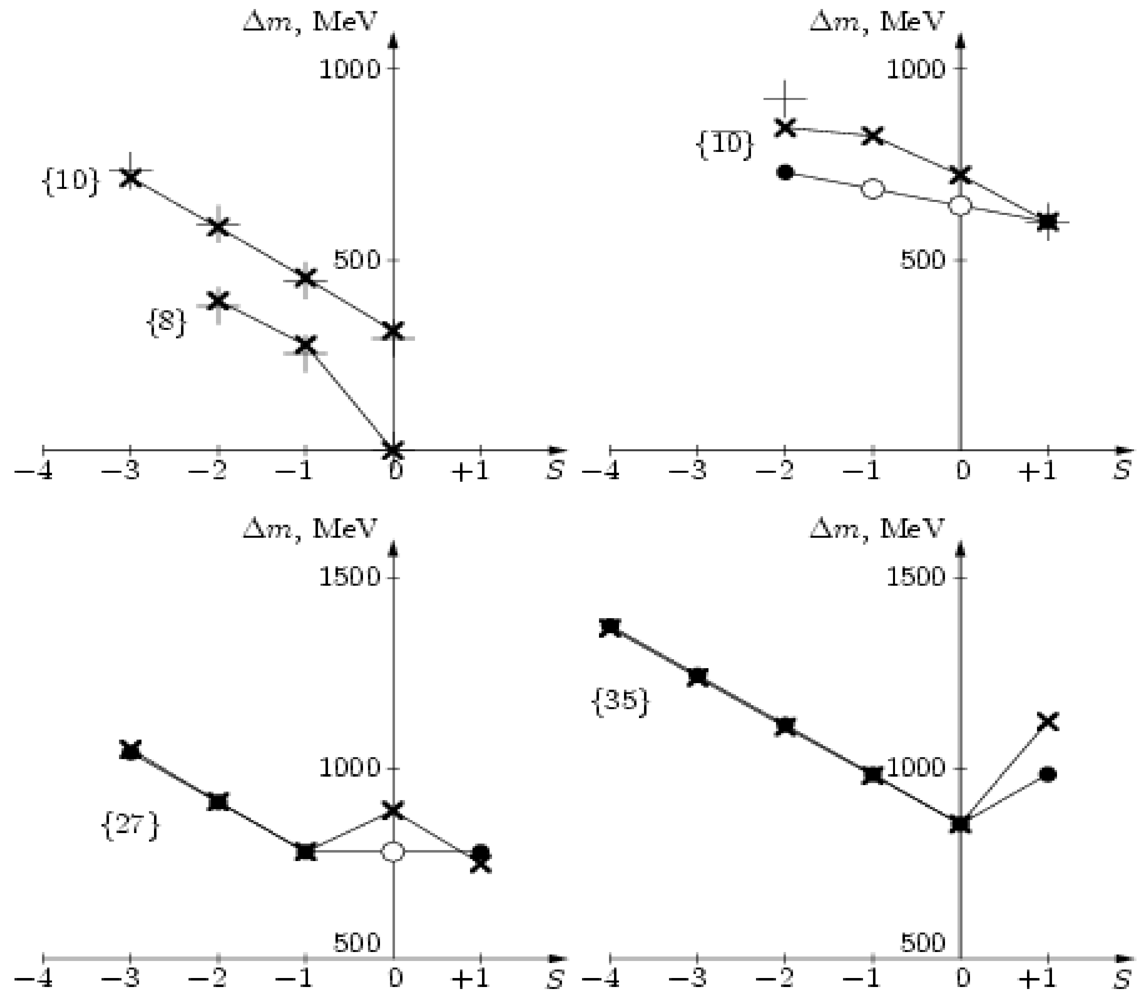
$$[m_s]_{27} \simeq 135\, Mev\ (117\, Mev),$$

and from 35-plet

$$[m_s]_{35} = 130\, Mev\ (114\, Mev),$$

$$[m_{\bar{s}}]_{35} \simeq 270\, Mev\ (295\, Mev).$$

Strong dependence of s-antiquark mass on the multiplet:
artefact of CSA, or it is physically significant ?



Location of baryon states - nonexotic and exotic - within multiplets of baryons. + show data, black circles - manifestly exotic states within simplistic quark model with the strange quark mass $m_s \simeq 130\ MeV$, empty circles - cryptoexotic states, ×- results of calculations within chiral soliton (rigid rotator) model (figure from *VBK and A.M.Shunderuk, Phys.Rev. D73, 094018, 2006*).

Diquarks mass difference estimates can be made roughly from CSA.

$[q_1 q_2]$ - singlet, $\bar{3}_F$ diquark ("good" d_0),
$(q_1 q_2)$ - triplet, 6_F "bad" d_1 (F.Wilczek).

Examples of wave functions of PQ-s in the diquark-diquark-antiquark picture by R.Jaffe and F.Wilczek:

$\Theta_0 \in \{\overline{10}\} \sim [ud][ud]\bar{s}$,

$\Phi/\Xi^{--}_{3/2} \in \{\overline{10}\} \sim [sd][sd]\bar{u}$,

It is not possible to built $\{27\}$ and $\{35\}$-plets from "good" diquarks, "bad" diquarks are needed:

$\Theta_1 \in \{27\} \sim (ud)[ud]\bar{s}$; $\Theta_2 \in \{35\} \sim (ud)(ud)\bar{s}$, Since "bad" diquark is heavier, this is obvious reason why Θ_1 is heavier than Θ_0, and Θ_2 is more heavy.

It seems to be natural to ascribe the difference of rotation energies for different multiplets to the difference of masses of "bad" and "good" diquarks.

From difference of {27}-plet and antidecuplet masses

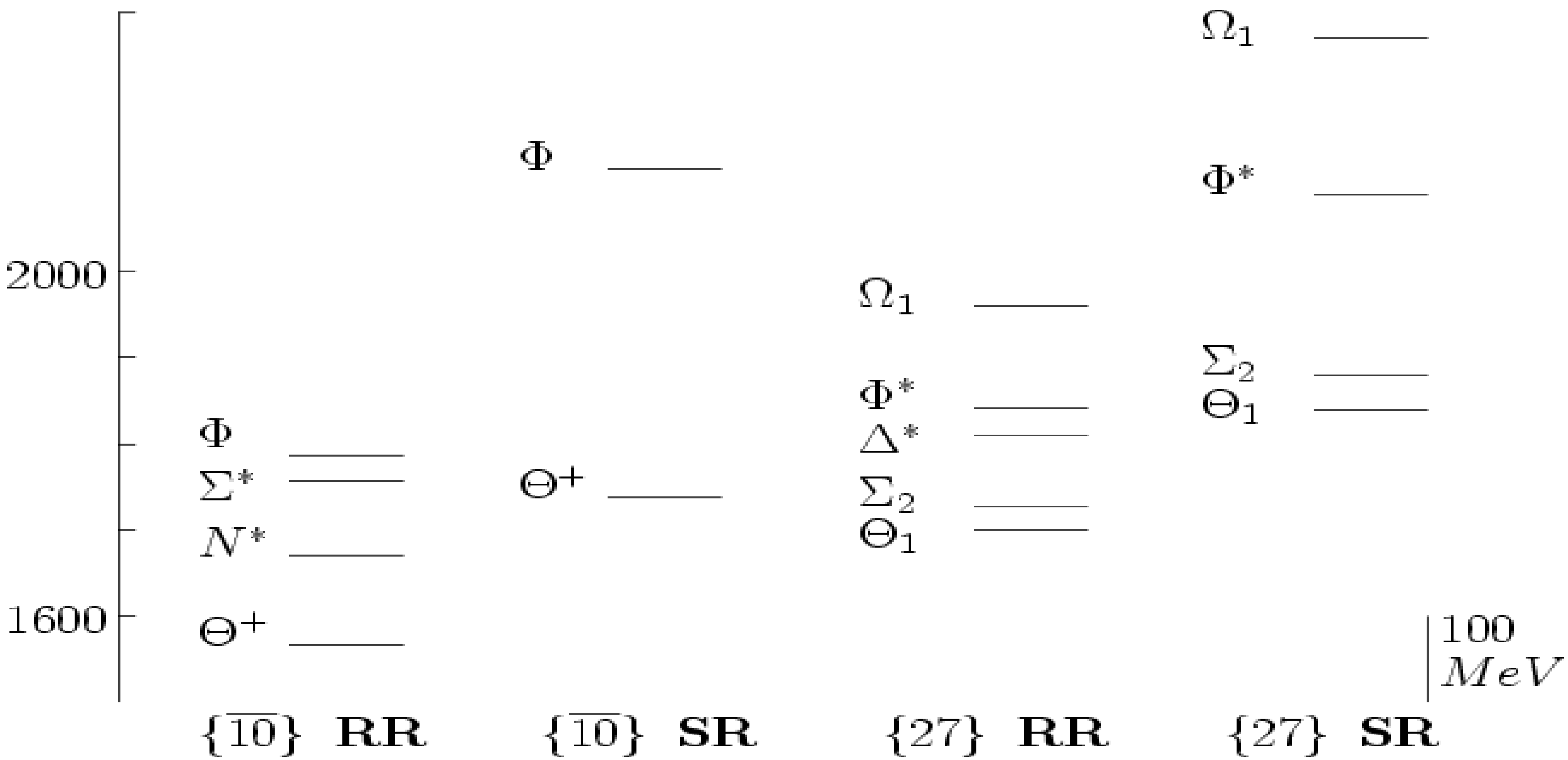
$$M(d_1) - M(d_0) \sim \frac{3}{2\Theta_\pi} - \frac{1}{2\Theta_K} \sim 100\,MeV.$$

from {35}-plet and {27}-plet mass difference

$$M(d_1) - M(d_0) \sim \frac{5}{2\Theta_\pi} - \frac{1}{2\Theta_K} \sim 250\,MeV.$$

Qualitatively seems to be OK, but this picture should be too naive. E.g., Interaction between diquarks may be important.

6. Rigid Rotator — Soft Rotator dilemma



Comparison of the rigid rotator (RR) and soft rotator (SR) models predictions for the masses of exotic baryons, antidecuplet and 27-plets. Not all states are shown for the SR model. The code for SR model was arranged by B.Schwesinger, H.Weigel (1992).

The lower index indicates the isospin of the state, e.g.

$$\Sigma_2 = |27, Y = 0, I = 2 >, \ \Omega_1 = |27, Y = -2, I = 1 > .$$

Static characteristics of skyrmions depend on ν - the angle of rotation into "strange" direction. For "strange", or kaonic inertia it is most important:

$$\Theta_K = \frac{1}{8}\int (1-c_f)\Big[F_K^2 - sin^2\nu(F_K^2 - F_\pi^2)(2-c_f)/2 + ...$$

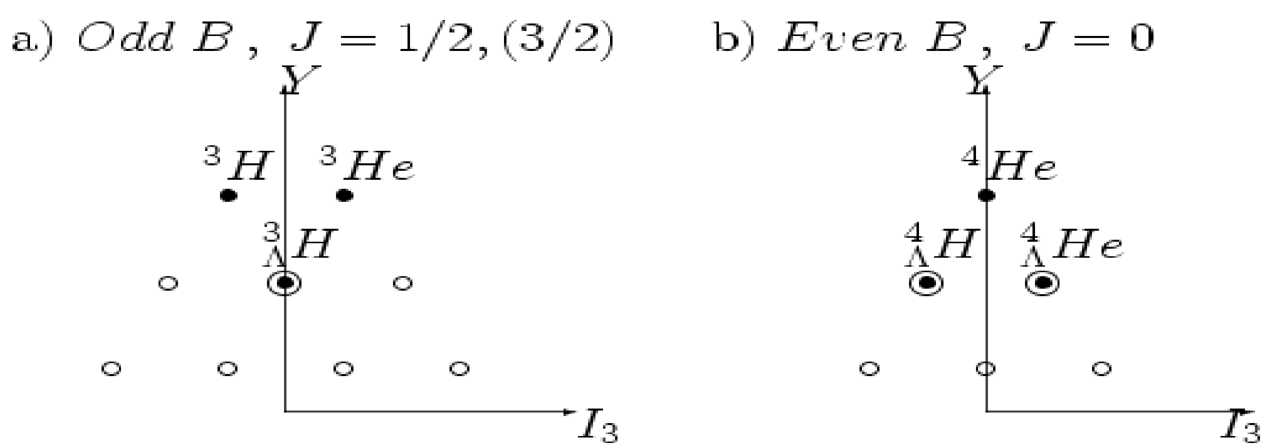
it is decreasing function of $sin^2\nu$. RRM corresponds to $\nu = 0$.

In the soft rotator model, opposite to rigid rotator, it is supposed that soliton is deformed under influence of FSB forces: static energy minimization is made at fixed value of ν. Dependence on ν of static characterstics of skyrmions is taken into account in the quantization procedure.

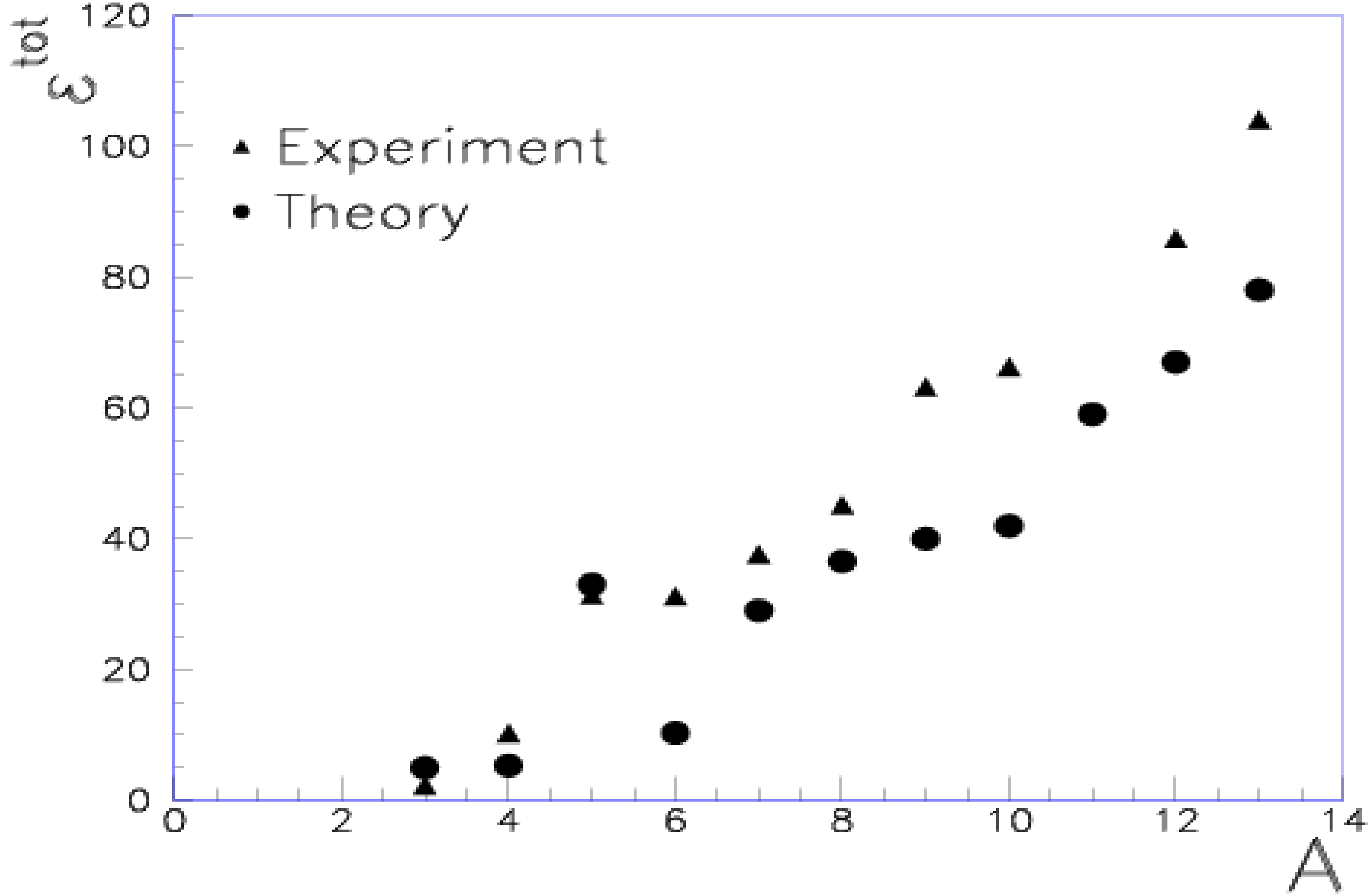
7. Multibaryons

Advantage of CSA: multibaryon states - nuclei, hypernuclei ... - can be considered on equal footing with B=1 case. The rational map approximation (Houghton, Manton, Sutcliffe, 1997) simplifies this work considerably. $\Theta_I \sim B$, $\Theta_J \sim B^2$ for $B \leq 20 - 30$. Some kind of the "bag model" is obtained, starting with effective lagrangian.

Ordinary nuclei and hypernuclei (ground states) should be ascribed to definite $SU(3)$ multiplets.



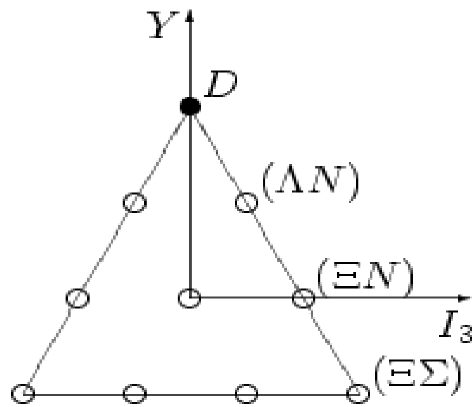
(a) The location of the isoscalar state (shown by double circle) with odd B-number and $|F| = 1$ in the upper part of the $(I_3 - Y)$ diagram.
(b) The same for isodoublet states with even B. The case of light hypernuclei $_\Lambda H$ and $_\Lambda He$ is presented as an example. The lower parts of diagrams with $Y \leq B - 3$ are not shown here. In a version of BSM it is possible to describe total binding energies of light hypernuclei in qualitative, even semiquantitative agreement with data.



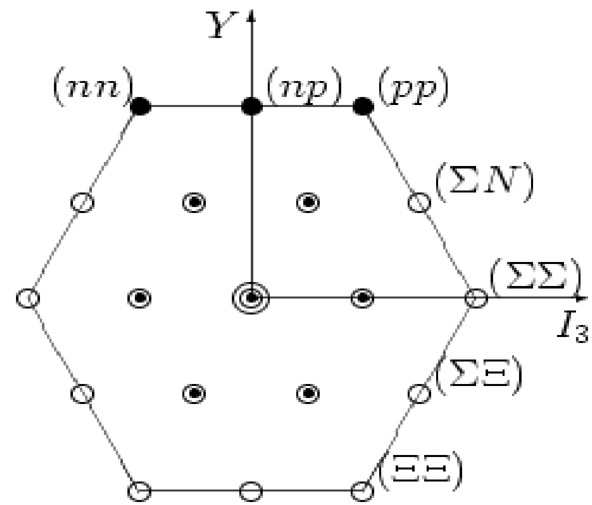
Total binding energies ϵ^{tot} (in MeV) of the $S = -1$ hypernuclei with atomic numbers up to 13. Full triangles - experimental data, full circles - contribution to ϵ^{tot} of multiskyrmions collective motion, within CSA.

Some rotational excitations of bound skyrmions can be interpreted as deeply bound $\bar{K}$-nuclear bound states.

$$\Delta E = \frac{J(J+1)}{2\Theta_J}$$



$B = 2,\ \{\overline{10}\},\ J = 1\,(2, ...)$



$B = 2,\ \{27\},\ J = 0\,(1, ...)$

$I_3 - Y$ diagrams of multiplets of dibaryons $B = 2,\ m = 0$. Virtual levels (scattering states) are shown in brackets, e.g. (ΛN) scattering state which appears as near threshold enhancement.

$J = 2^+$ excited states have energy by $\sim 2/\Theta_J$ greater than $\overline{10}$. The state with $S = -1$, $I = 1$, $J^P = 2^+$ can be interpreted as NNK with binding energy $60 - 100\ Mev$.

$J = 1$ states. For the $B = 2$ $\{27\}$-plet $J = 1$ states have energy by $1/\Theta_J$ greater than $J = 0$ ground states.

Rotational excitations have additional energy

$$\Delta E = \frac{J(J+1)}{2\Theta_J}.$$

The orbital inertia grows fast with increasing baryon (atomic) number, $\Theta_J \sim B^p$, p is between 1 and 2. By this reason the number of rotational states becomes large for large baryon numbers. More detailed investigations are necessary.

Summary and Conclusions

$*$ Expansion parameter in $1/N_c$ is large for the case of spectrum, extrapolation to real world is not possible in this way.

$*$ Rigid (soft as well) rotator and bound state models coincide in the first order of $1/N_c$, but *differ* in the next orders.

$*$ Configuration mixing is important, according to RRM.

$*$ Transition to Soft Rotator Model from RRM may be crucial, leading to the increase of masses.

$*$ There is correspondence of chiral solitons (RRM) and quark model predictions for pentaquarks spectra in negative S sector of $\{27\}$ and $\{35\}$ plets: the effective mass of strange quark is about $135 - 130\, MeV$, slightly smaller for $\{35\}$.

$*$ For positive strangeness components the link between CSM and QM requires strong dependence of effective $\bar{s}$ mass on particular $SU(3)$ multiplet. Config. mixing pushes spectra towards *simplistic* model - nice property, but reasons are not clear.

$*$ Diquarks mass difference estimates from CSA seem to be reasonable.

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* Chiral soliton models, based on few principles and ingredients incorporated in effective lagrangian, allow to describe qualitatively, in some cases even quantitatively, various chracteristics of baryons and nuclei - from ordinary $(S = 0)$ nuclei to known hypernuclei.

* This suggests that predictions of pentaquark states should be considered seriously. Existence of PQ by itself is without any doubt, although very narrow PQ may not exist. Wide, even very wide PQ should exist. Searches for PQ-s remain an actual task.

* There are, however, problems when one tries to project results of the CSA on the Quark Models: strong dependence of strange antiquark mass on the $SU(3)$ multiplet; difference of masses of "bad" and "good" diquarks is not unique in naive picture, at least.

* In view of theoretical uncertainties, experimental investigations could play decisive role.
Future studies at J-PARC are of importance at this point: *great chance to shed more light on the puzzles of baryon spectroscopy.*