Possible Study of GPDs at J-PARC

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NP08, Mito
March 6, 2008

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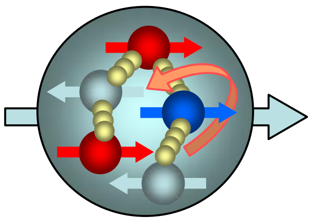
Introduction

- **Nucleon's spin sum rule:**

$$\frac{1}{2}=\frac{1}{2}\underbrace{\left(\Delta u_v+\Delta d_v+\Delta q_{sea}\right)}_{\Delta\Sigma}+L_q+\Delta G+L_g$$



  - Spin carried by quarks: $\Delta\Sigma$~0.2-0.3
  - Spin carried by gluons may be small. (cf. RHIC data)

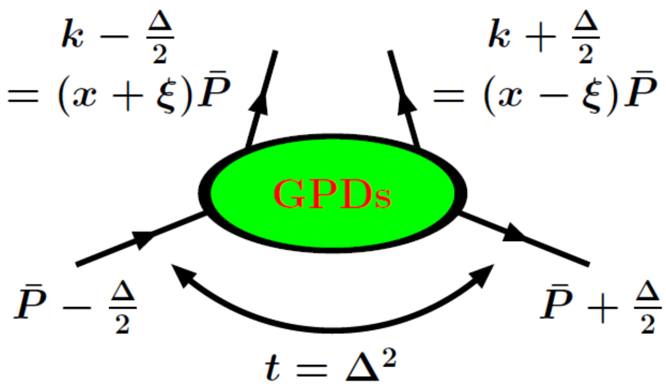
- **Orbital angular momentum should be studied**
  - Generalized parton distributions (GPDs) play an important role.

    Can access to quark total angular momentum
  - From DVCS and/or hard exclusive meson production

We discuss a possibility to extract information about GPDs from J-PARC experiment with 30~50 GeV primary proton beam.

Generalized Parton Distribution

- **GPDs are defined as light-cone correlation of off-forward M.E.**



Generalized Bjorken variables: ***x***

$$k^+ = x\bar{P}^+, \quad \bar{P} = (p+p')/2$$

Skewdness parameter: ξ

$$\Delta^+ = -2\xi\bar{P}^+$$

Momentum transfer squared: ***t***

$$t = (P'-P)^2 = \Delta^2$$

[Ji, 1997]

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}\left(-\frac{\lambda}{2}n\right)[\not{n} + \not{n}\gamma_5]\psi\left(\frac{\lambda}{2}n\right) | P \rangle$$

$$= \left[H(x,\xi,t)\bar{u}(P')\not{n}u(P) + E(x,\xi,t)\bar{u}(P')\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}u(P) \right] \quad \textbf{(Unpolarized)}$$

$$+ \left[\tilde{H}(x,\xi,t)\bar{u}(P')\not{n}\gamma_5 u(P) + \tilde{E}(x,\xi,t)\bar{u}(P')\frac{\gamma_5(n\cdot\Delta)}{2M}u(P) \right] \quad \textbf{(Polarized)}$$

Helicity-conserve (H, $\tilde{H}$) **Helicity-flip** (E, $\tilde{E}$)

Features of GPDs

- **In forward limit** ➡ **usual PDFs**

$$H(x,\xi,t)\big|_{\xi=t=0} = q(x), \qquad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta q(x)$$

There is no analog in E and $\tilde{E}$.

- **First moment of GPDs** ➡ **Nucleon form factors**

$$\int dx\, H(x,\xi,t) = F_1(t), \qquad \int dx\, E(x,\xi,t) = F_2(t),$$

$$\int dx\, \tilde{H}(x,\xi,t) = G_A(t), \qquad \int dx\, \tilde{E}(x,\xi,t) = G_P(t)$$

- **Second moment of GPDs** ➡ **Quark angular momentum**

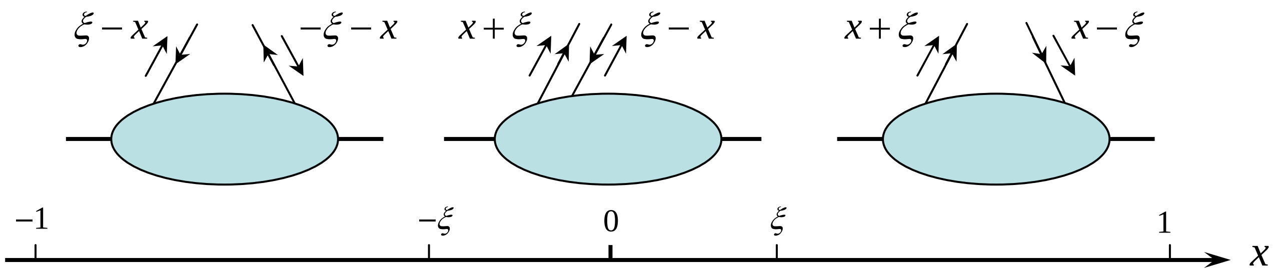
$$J^i = \frac{1}{2}\varepsilon^{ijk}\int d^3z\, M^{0jk}, \quad M^{\mu\nu\lambda} = z^\nu T^{\mu\lambda} - z^\lambda T^{\mu\nu}, \quad T^{\mu\nu}: \text{energy-momentum tensor}$$

$$J^i_{q,g} = \langle P' | \int d^3z\, (\vec{z}\times\vec{T}_{q,g})^i | P \rangle$$

Ji's sum rule: $J_q = \frac{1}{2}\int dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right]$; $\quad J_q = \sum_q \frac{1}{2}\Delta q + L_q$

Physical Interpretation in x and ξ

- **Parton interpretation of GPDs in different x-intervals**



– **(1) Quark distribution** $\xi < x < 1 \quad (x+\xi > 0,\ x-\xi > 0)$

Emission of quark with momentum fraction $x+\xi$
Absorption of quark with momentum fraction x-ξ

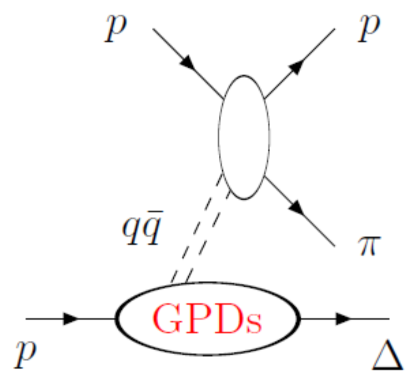
– **(2) Meson distribution amplitude** $-\xi < x < \xi \quad (x+\xi > 0,\ x-\xi < 0)$

Emission of quark with momentum fraction $x+\xi$
Emission of antiquark with momentum fraction ξ-x

– **(3) Antiquark distribution** $-1 < x < \xi \quad (x+\xi < 0,\ x-\xi < 0)$

Emission of antiquark with momentum fraction ξ-x
Absorption of antiquark with momentum fraction -x-ξ

Exclusive Reaction: $NN \rightarrow N\pi\Delta$



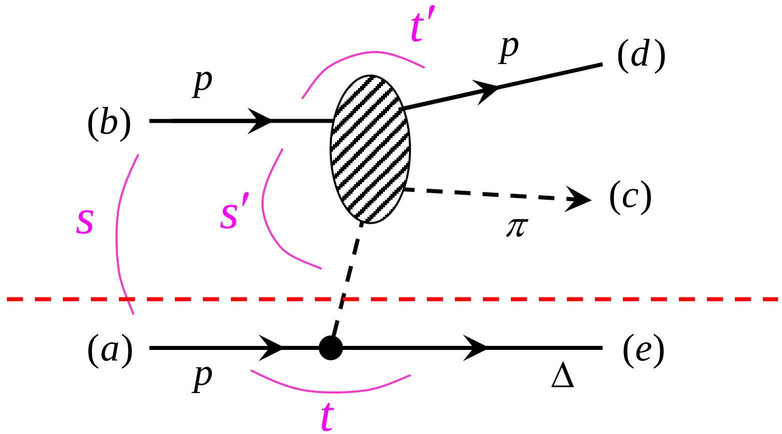
- **High energy exclusive reaction: $pp \rightarrow p\pi\Delta$**
  - $p \rightarrow p$: large angle scattering (t': large)
  - $p \rightarrow \Delta$: forward region (t: small)
- **$N \rightarrow \Delta$ transition GPDs is inserted**
  - $q\bar{q}$ exchange process dominates in $t = m_\pi^2$
  - Large N_c relation between $N \rightarrow \Delta$ and $N \rightarrow N$ GPDs
  - No study of GPDs in pp reaction

$$d\sigma = \frac{dLips}{F} \left| M_{pp \rightarrow p\pi\Delta} \right|^2 \propto \int_{-1}^{1} dx\, F_{p \rightarrow \Delta}(x, \xi, t) \frac{d\sigma_{\pi p \rightarrow \pi p}(s', t')}{dt'} dt'$$

- **This process can be observed by J-PARC experiment with primary proton beam.**

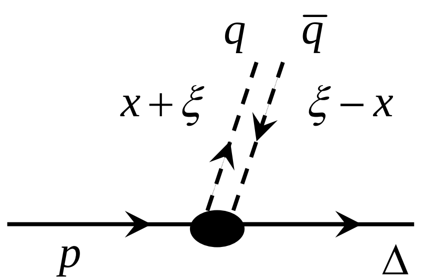
Factorization

- **Factorization is assumed as below.**



$$s=(p_a+p_b)^2,\quad t=(p_e-p_a)^2\equiv\Delta^2$$
$$s'=(p_\pi+p_b)^2,\quad t'=(p_d-p_b)^2;$$
$$p_\pi=p_a-p_e\equiv-\Delta$$

Assuming factorization



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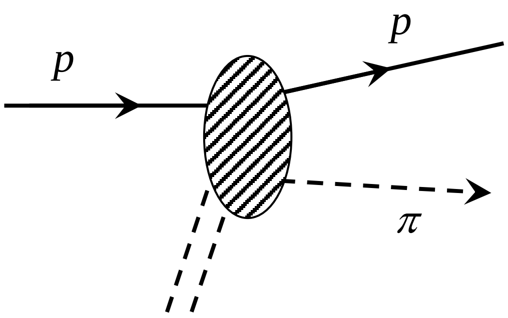


p→Δ transition GPDs (soft)

Subprocess cross section (hard)

$N \rightarrow \Delta$ Transition GPDs

- **Helicity-independent $p \rightarrow \Delta^+$ transition GPDs** Frankfurt et al, '98, '00.

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle \Delta, p_e \left| \bar{\psi}\left(-\frac{\lambda}{2}n\right) \not{n} \tau^3 \psi\left(\frac{\lambda}{2}n\right) \right| N, p_a \right\rangle$$

$$= \sqrt{\frac{2}{3}} \bar{\psi}_\Delta^\mu(p_e) \left[H_M(x,\xi,t) K_{\mu\nu}^M n^\nu + H_E(x,\xi,t) K_{\mu\nu}^E n^\nu + H_C(x,\xi,t) K_{\mu\nu}^C n^\nu \right] \psi_N(p_a)$$

$$\int_{-1}^{1} H_{M,E,C}(x,\xi,t) dx = 2G_{M,E,C}^*(t)$$ **: transition form factor**

M **: magnetic dipole**

E **: electric quadrupole**

C **: Coloumb quadrupole**

- **Helicity-dependent $p \rightarrow \Delta^+$ transition GPDs**

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle \Delta, p_e \left| \bar{\psi}\left(-\frac{\lambda}{2}n\right) \not{n} \gamma^5 \tau^3 \psi\left(\frac{\lambda}{2}n\right) \right| N, p_a \right\rangle$$

$$= \bar{\psi}_\Delta^\mu(p_e) \left[\tilde{H}_1(x,\xi,t) n_\mu + \tilde{H}_2(x,\xi,t) \frac{\Delta_\mu (n \cdot \Delta)}{m_N^2} + \tilde{H}_3(x,\xi,t) \frac{n_\mu \not{\Delta} - \Delta_\mu \not{n}}{m_N} + \tilde{H}_4(x,\xi,t) \frac{\bar{P} \cdot \Delta n_\mu - 2\Delta_\mu}{m_N^2} \right] \psi_N(p_a)$$

Large N_c Relation

- **large N_c relations (LO in $1/N_c$ expansion)** Goeke et al, 2001.

$$H_M(x,\xi,t) = \frac{2}{\sqrt{3}}\left[E^u(x,\xi,t) - E^d(x,\xi,t)\right]$$
$$\tilde{H}_1(x,\xi,t) = \sqrt{3}\left[\tilde{H}^u(x,\xi,t) - \tilde{H}^d(x,\xi,t)\right]$$
$$\tilde{H}_2(x,\xi,t) = \frac{\sqrt{3}}{4}\left[\tilde{E}^u(x,\xi,t) - \tilde{E}^d(x,\xi,t)\right]$$
$$H_E(x,\xi,t) = H_C(x,\xi,t) = \tilde{H}_3(x,\xi,t) = \tilde{H}_4(x,\xi,t) = 0$$

- $H_M(x,\xi,t)$ **in the large N_c limit**
  - Isovector part of the angular momentum of nucleon by quarks

$$\lim_{t\to 0, N_c\to\infty} \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \frac{2}{\sqrt{3}}\left[2(J^u - J^d) - M_2^u + M_2^d\right]$$
$$: M_2^q \equiv \int_0^1 dx\, x\left[q(x) + \bar{q}(x)\right]$$

Transition Amplitude

- **Helicity-independent part:**

$$M^V_{p\to\Delta} = \sqrt{\frac{2}{3}}\int_{-1}^{1} dx\,\bar{\psi}^\mu_\Delta(p_e) H_M(x,\xi,t) K^M_{\mu\nu} n^\nu \psi_N(p_a)$$

$$\sum_{\lambda_N,\lambda_\Delta}\left|M^V_{p\to\Delta}\right|^2 = \frac{2}{3}\left[\int_{-1}^{1} dx H_M(x,\xi,t)\right]^2 \mathbf{Tr}\left[\sum_{\lambda_\Delta}\psi^\alpha_\Delta(p_e)\bar{\psi}^\mu_\Delta(p_e)\cdot K^M_{\mu\nu}n^\nu \cdot \sum_{\lambda_N}\psi_N(p_a)\bar{\psi}_N(p_a)\cdot K^M_{\alpha\beta}n^\beta\right]$$

$$\sum_{\lambda_N}\psi_N(p_a)\bar{\psi}_N(p_a) = (\not{p}_a - m_N)$$

$$\sum_{\lambda_\Delta}\psi^\alpha_\Delta(p_e)\bar{\psi}^\mu_\Delta(p_e) = (\not{p}_e - m_\Delta)\left(-g^{\alpha\mu} + \frac{1}{3}\gamma^\alpha\gamma^\mu - \frac{p_e^\alpha\gamma^\mu - p_e^\mu\gamma^\alpha}{3m_\Delta} + \frac{2p_e^\alpha p_e^\mu}{3m_\Delta^2}\right)$$

$$K^M_{\mu\nu} = -i\frac{3(m_\Delta + m_N)}{2m_N\left[(m_\Delta+m_N)^2 - t\right]}\varepsilon_{\mu\nu\rho\sigma}\bar{P}^\rho\Delta^\sigma$$

$$C_M(\xi,t) \equiv \mathbf{Tr}[\cdots] = \frac{3(m_\Delta+m_N)^2\left[4t - 4(m_\Delta^2+m_N^2) + m_\Delta m_N\right]\left[t(1-4\xi^2) + 4\xi(m_\Delta^2 - m_N^2) + 8\xi^2(m_\Delta^2+m_N^2)\right]}{16m_N^2\left[(m_\Delta+m_N)^2 - t\right]^2}$$

Transition Amplitude

- **Helicity-dependent part:**

$$M^A_{p\to\Delta} = \int_{-1}^{1} dx \bar{\psi}^\mu_\Delta(p_e)\left[\tilde{H}_1(x,\xi,t)n_\mu + \tilde{H}_2(x,\xi,t)\frac{\Delta_\mu(n\cdot\Delta)}{m_N^2}\right]\psi_N(p_a)$$

pion-pole: $\tilde{H}_\pi$ $$M^{\pi-pole}_{p\to\Delta} = \frac{g_A\sqrt{3}}{m_\pi^2 - t}\bar{\psi}^\mu_\Delta(p_e)\Delta_\mu\psi_N(p_a)$$

$$\therefore M^A_{p\to\Delta} = \int_{-1}^{1} dx \bar{\psi}^\mu_\Delta(p_e)\left[\tilde{H}_1(x,\xi,t)n_\mu + \left(\tilde{H}_2(x,\xi,t)\frac{n\cdot\Delta}{m_N^2} + \tilde{H}_\pi(x,\xi,t)\right)\Delta_\mu\right]\psi_N(p_a)$$

$$\sum_{\lambda_N,\lambda_\Delta}\left|M^A_{p\to\Delta}\right|^2 = \int_{-1}^{1} dx \int_{-1}^{1} dx' \Big[\tilde{H}_1(x,\xi,t)\tilde{H}_1(x',\xi,t)C_1(\xi,t) + \tilde{H}_2(x,\xi,t)\tilde{H}_2(x',\xi,t)C_2(\xi,t)$$
$$+ \tilde{H}_1(x,\xi,t)\tilde{H}_2(x',\xi,t)C_{12}(\xi,t) + \tilde{H}_\pi(x,\xi,t)\tilde{H}_\pi(x',\xi,t)C_\pi(\xi,t)$$
$$+ \tilde{H}_1(x,\xi,t)\tilde{H}_\pi(x',\xi,t)C_{1\pi}(\xi,t) + \tilde{H}_2(x,\xi,t)\tilde{H}_\pi(x',\xi,t)C_{2\pi}(\xi,t)\Big]$$

$C_{ij}(\xi,t)$: **cofficient function**

Transition Amplitude

$$C_1(\xi,t)=\frac{1}{6m_\Delta^2}(2\xi-1)^2(2m_\Delta^2+m_\Delta m_N+m_N^2-2t)$$

$$C_2(\xi,t)=\frac{2\xi^2}{3m_\Delta^2 m_N^4}\Big\{2(m_\Delta^2+m_N^2)(m_\Delta^2-m_N^2)^2+m_\Delta m_N\left[t+(m_\Delta^2-m_N^2)\right]^2$$
$$+6t(m_\Delta^2+m_N^2)\left[t+(m_\Delta^2-m_N^2)\right]-2t(t^2-m_\Delta^3 m_N+2m_\Delta^2 m_N^2)\Big\}$$

$$C_{12}(\xi,t)=\frac{2\xi}{3m_\Delta^2 m_N^2}\Big\{(t-m_\Delta^2)\left[2(1-2\xi)(t-m_\Delta^2)-m_\Delta m_N(1-2\xi)+16\xi m_\Delta^2\right]$$
$$-m_\Delta^3\left[6\xi(2m_\Delta-m_N)-2m_\Delta+m_N\right]\Big\}$$

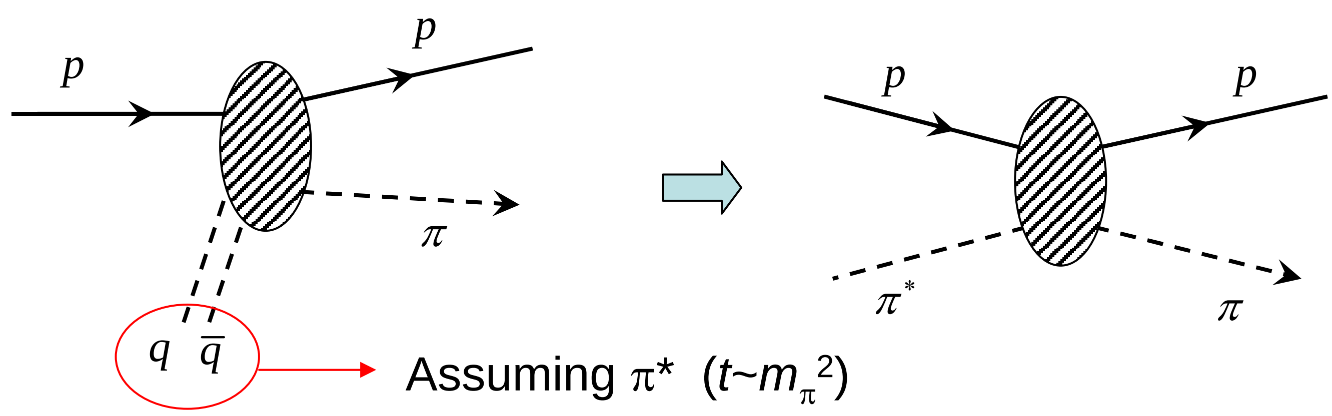
$$C_\pi(\xi,t)=\frac{m_N^4}{4\xi^2}C_2(\xi,t),\qquad C_{1\pi}(\xi,t)=-\frac{m_N^2}{2\xi}C_{12}(\xi,t),\qquad C_{2\pi}(\xi,t)=\frac{m_N^2}{\xi}C_2(\xi,t)$$

- **Total amplitude**

$$\sum_{\lambda_N,\lambda_\Delta}\left|M_{p\to\Delta}\right|^2=\sum_{\lambda_N,\lambda_\Delta}\left(\left|M_{p\to\Delta}^V\right|^2+\left|M_{p\to\Delta}^A\right|^2\right)$$

πN → πN Scattering

- πN→πN cross section



Assuming π* ($t \sim m_\pi^2$)

The cross section is parameterized as

$$\frac{d\sigma}{dt'}(\pi p \to \pi p) = \frac{1}{s'^{n-2}}\left[a + c[t' - t'(90°)]^2\right]$$

White et al, PRD49, 58 (1994).

n : **number of quarks,** a, c : **parameter**

$$s' = (p_\pi + p_b)^2, \quad t' = (p_b - p_d)^2; \quad p_\pi = p_a - p_e \equiv -\Delta$$

Parameters a and c are obtained by fitting the experimental data.

Cross Section: *pp* → *p*πΔ

- *pp* → *p*πΔ cross section

$$d\sigma = \frac{1}{4\sqrt{(p_a\cdot p_b)^2 - m_N^4}} \frac{d^3p_c}{(2\pi)^3 2E_c} \frac{d^3p_d}{(2\pi)^3 2E_d} \frac{d^3p_e}{(2\pi)^3 2E_e} \sum_{\lambda_N,\lambda_\Delta} \left| M_{pp\to p\pi\Delta} \right|^2 \times (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d - p_e)$$

$$\frac{d\sigma}{dtdt'dE_\Delta} = \frac{\sqrt{(s'-t+m_N^2)^2 - 4tm_N^2}}{2(2\pi)^2\sqrt{s}(s-4m_N^2)} \frac{d\sigma_{\pi p\to\pi p}(s',t')}{dt'} \times \sum_{i,j}\int_{-1}^{1}dx\int_{-1}^{1}dx'\, C_{ij}(\xi,t) F_i(x,\xi,t) F_j(x',\xi,t)$$

C_i : **cofficient function**

F_i : **GPDs**

- **Parametrization of GPDs**
  - use ξ-independent ansatz [Vanderhaeghen et al, PRD60, 094017 (1999).]

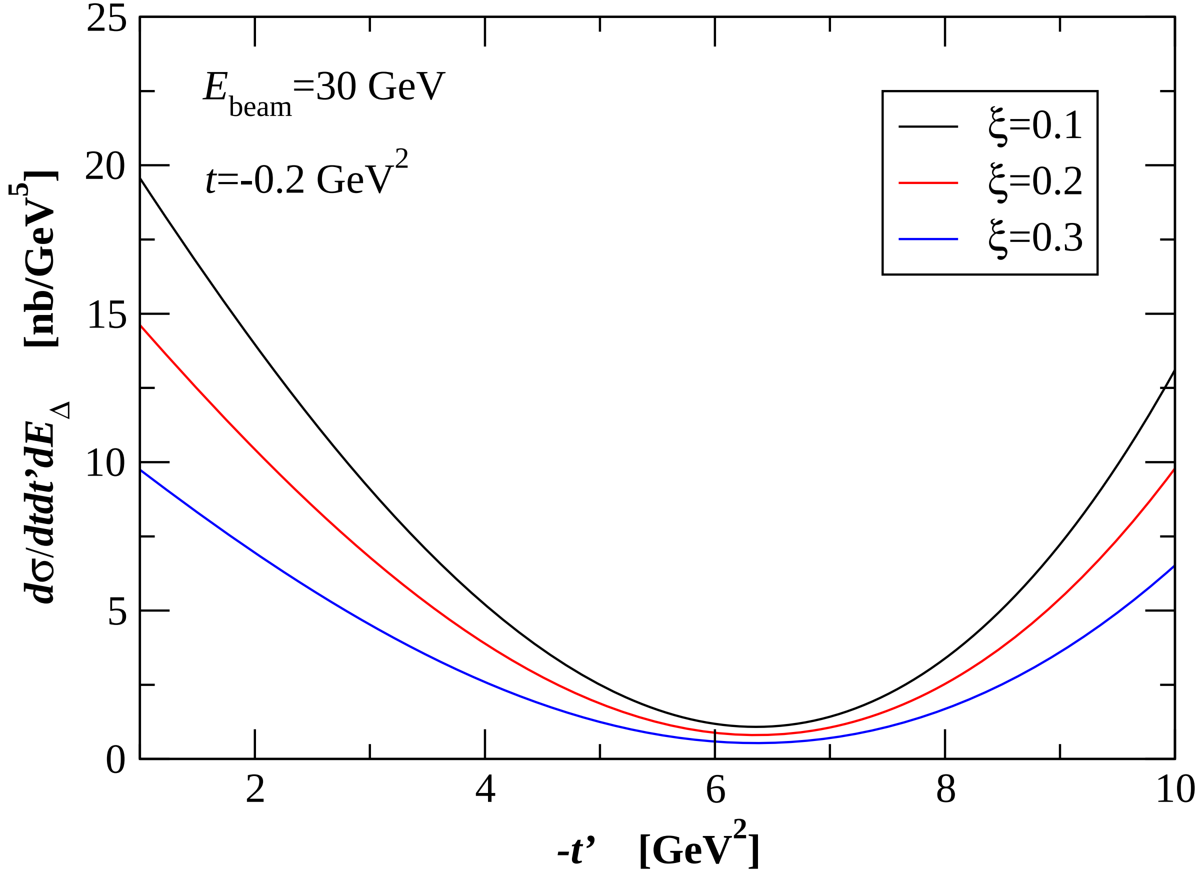
$$E^u(x,\xi,t) = u(x)F_2^u(t)/2 \qquad \tilde{H}^u(x,\xi,t) = \Delta u(x) g_A^u(t)/g_A^u(0)$$

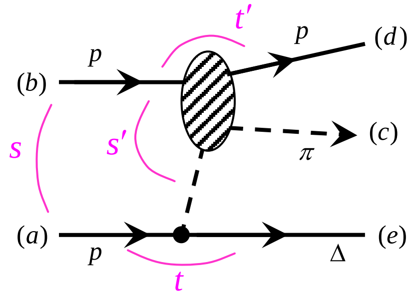
$$E^d(x,\xi,t) = d(x)F_2^u(t) \qquad \tilde{H}^d(x,\xi,t) = \Delta d(x) g_A^d(t)/g_A^d(0)$$

Cteq6 for $q(x)$

AAC03 for $\Delta q(x)$

Numerical Results





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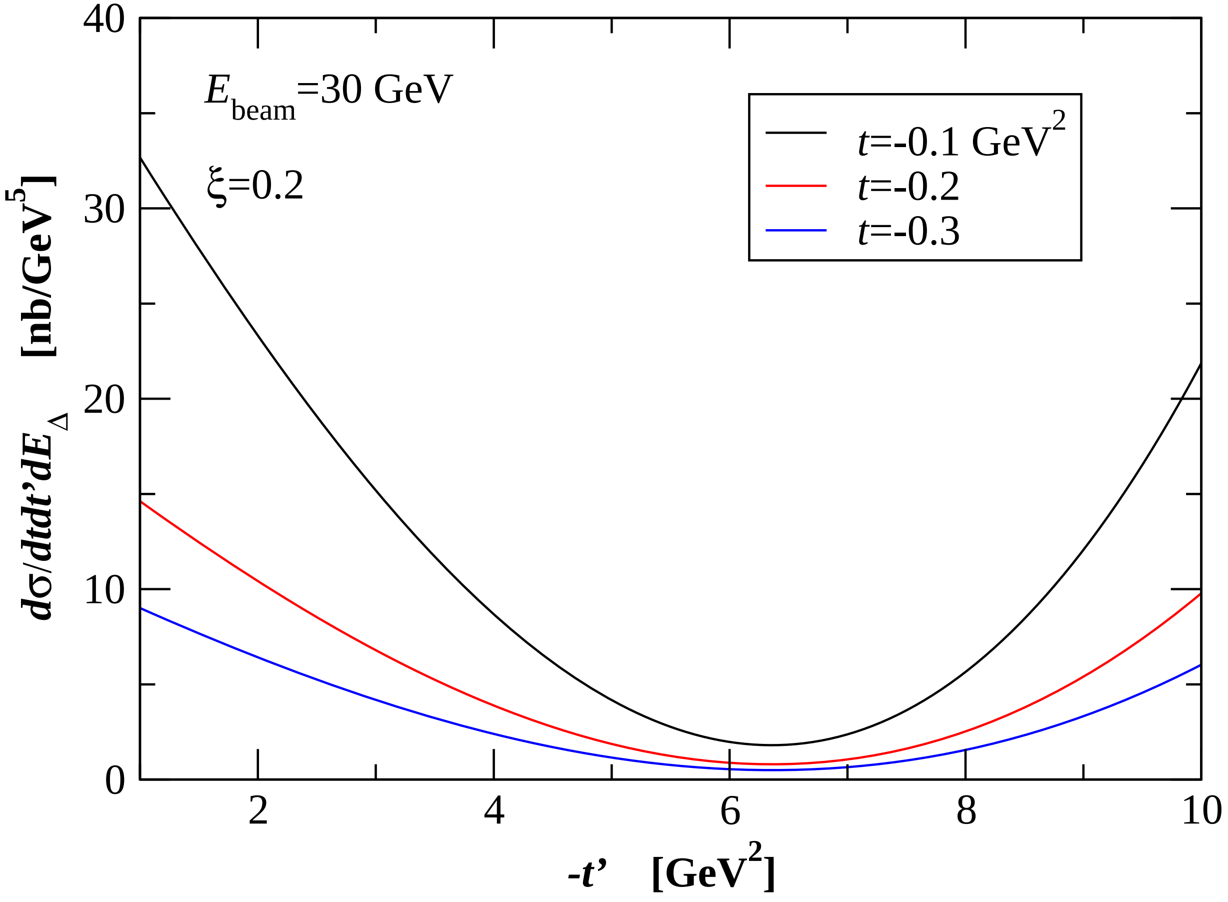
Numerical Results

• *p*



Summary

- **GPDs are a key issue to study the quark angular momentum.**

$$J_q = \frac{1}{2}\int dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right]$$

  - Experiments are under going in DVCS/HEMP processes.

- **The process $pp \rightarrow p\pi\Delta$ was investigated.**
  - Assuming factorization and $q\bar{q}$ dominance
  - Extract information about isovector part of quark angular momentum
  - The cross section is measurable at J-PARC

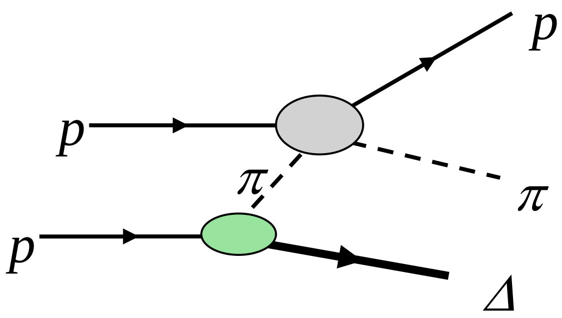
Outlook:

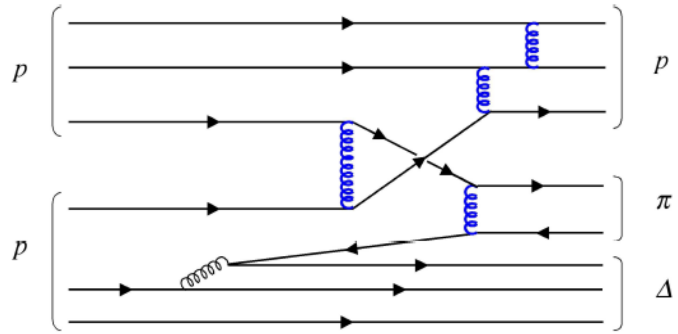
- **Need more reliable calculation** **(parametrization of GPDs, kinematics,,,,)**
- **Other processes might be also challenging.**

Back Up

Counting Rule

- **Leading contribution**





- **Subleading contribution**